ASTRONOMY

A REVISION OF
YOUNG'S MANUAL OF ASTRONOMY

I
THE SOLAR SYSTEM

BY

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In dealing with such subjects as the constitution and evolution of the stars (concerning which theories are in a state of very active flux) the attempt has been made to present the situation as it appears to the writers at the "epoch" of completion of the manuscript, in 1926. Considerable changes may be necessary within a few years.

The whole manuscript of the book has been read by at least two of the authors, and usually by all three, and every effort has been made to avoid errors of statement and to obtain the best available values of numerical data. It cannot be hoped, however, that such efforts have been wholly successful, and information concerning any errors which may be detected will be welcomed. Professor Young's statements concerning the earlier history of the science have usually been accepted without fresh investigation.

New material for illustrations has been generously supplied by numerous friends in this country and abroad. Acknowledgments are gratefully made to the directors, and to many other members of the staffs, at the observatories at Greenwich, Heidelberg, and Victoria (British Columbia); also at the Mt. Wilson, Lowell, Lick, Yerkes, Harvard, and Yale observatories; to the United States Navy, the Carnegie Institution, the American Museum of Natural History, and the *Scientific American*; likewise to the late Professor Barnard, to Professors Stebbins and Slocum, Dr. Benjamin Boss, Mr. Donald B. MacMillan, Mr. D. M. Barringer, and Mr. Howard Russell Butler.

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I

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INTRODUCTION

1. **Astronomy,** as is indicated by the Greek roots of the word (*ἄστρον, νόμος*), is the science which treats of the heavenly bodies. It considers (1) their motions, both real and apparent, and the laws which govern those motions; (2) their forms, dimensions, masses, and surface features; (3) their nature, constitution, and physical condition; (4) the effects which they produce upon one another by their attractions and radiations; (5) their probable past history and future development.

As we look up at night we see in all directions the countless stars, and, conspicuous among them and looking like stars, though very different in their real nature, are scattered a few planets. Here and there appear faintly shining clouds of light, — the Milky Way, nebulæ, and possibly a comet. Most striking of all, if it happens to be in the heavens at the time, though really the most insignificant of all, is the moon. By day the sun alone is visible, flooding the air with its light and hiding the other objects from the unaided eye, but not all of them from the telescope.

The bodies thus seen from the earth are the heavenly bodies. The first great advance of modern science was the recognition that the earth itself should be counted among these. The earth, like most of the others, is a globe, whirling on its axis and moving swiftly through space, although on its surface we are wholly unconscious of the motion because of its perfect steadiness. Most of the heavenly bodies are so far away that their motions can be detected only by careful observation.

feeble light that reaches the earth through the depths of space. Starlight is a very precious thing to the astronomer, and he spares neither labor nor cost in devising instruments for collecting and analyzing it. His quantitative observations have attained a precision, both in direct measurement and in the detection of hidden sources of error, that is rarely equaled in any other science. In the discussion and interpretation of his observations he continually employs mathematical analysis, often of the most advanced type, and utilizes freely the latest results and conclusions of physics and chemistry.

4. Relation of Astronomy to other Sciences. Thus there is no sharp boundary between astronomy and the other physical sciences. It is so intimately related to physics that it is often quite impracticable to say whether a given piece of work should be regarded as belonging more to one science than to the other. Its relation to mathematics is almost as close. In problems concerning the formation and constitution of the earth, astronomy overlaps the field of geology; and in questions dealing with the structure of atoms it comes in touch with chemistry.

Points of contact with the natural sciences are fewer; but in discussions of the length of time during which the earth has been habitable, and the possible habitability of other planets, astronomy meets biology, while in the consideration of those errors of observation which are personal to the observer it utilizes principles of physiology and psychology.

5. Branches of Astronomy. Astronomy is conventionally divided into several branches.

(1) *Practical astronomy* deals with the field of observation, — the design and use of astronomical instruments, the methods of observing, the elimination of errors, and the deduction of the data employed in other branches of astronomy. It is quite as much an art as it is a science.

(2) *Astronomy of position*, or *astrometry*, treats of the geometrical relations of the heavenly bodies, their positions, distances, dimensions, and surface markings, and their real and apparent motions. A subdivision, *spherical astronomy*, deals with their apparent positions and motions on the background of the sky (celestial sphere).

The methods of nearly all such economically valuable observations, however, were discovered long ago. The end and object of present-day astronomy is chiefly knowledge pursued for its own sake. This no more needs defense than the pursuit of art; yet the most abstract astronomical investigations may in time have practical value, through their influence on the other sciences. Thus, astronomical studies undertaken for the sake of pure knowledge led, in the seventeenth century, to the discovery of the laws of dynamics and to the invention of the calculus, and so helped to lay the foundation for all later advances in physics and engineering. Very recently astronomy has had much to do with the development and testing of the important physical theory of relativity.

7. Value in Education. The student of astronomy, therefore, must expect his chief profit to be intellectual. Exactness of thought and expression are enforced by the precise character of many of the necessary discussions, and the taking of astronomical observations is a particularly instructive training in carefulness and accuracy. The solution of problems apparently hopeless cannot fail to impress the mind; the spectacle of simple law working out the most far-reaching results stimulates the imagination; and the beauty and grandeur of the subjects presented gratify the poetic sense.

For the advanced student there is no field in which it is possible sooner to get out to the front line of scientific advance and to learn how fresh territory is being won in a very active sector. Indeed, the number of objects that will repay observation is so great, and the opportunities for elementary calculation are so considerable, that undergraduate students, and amateurs without university training, have made and are making genuine contributions to the advance of astronomical knowledge.

8. Public Interest. Astronomy is the oldest of the sciences. Such striking phenomena as the rising and setting of the sun and moon, and the phases of the latter, must have been recognized as regular in the infancy of the race. Some of the earliest of all existing records relate to astronomical subjects, such as eclipses and the positions of the planets; and they are often of great value to the historian, since their dates can be accurately calculated.

appreciation of these facts cannot fail to possess an important influence in determining the attitude of the contemplative student toward such problems of philosophy as man's obligations to future generations, his place in the universe, and his relation to the Power which is behind it.

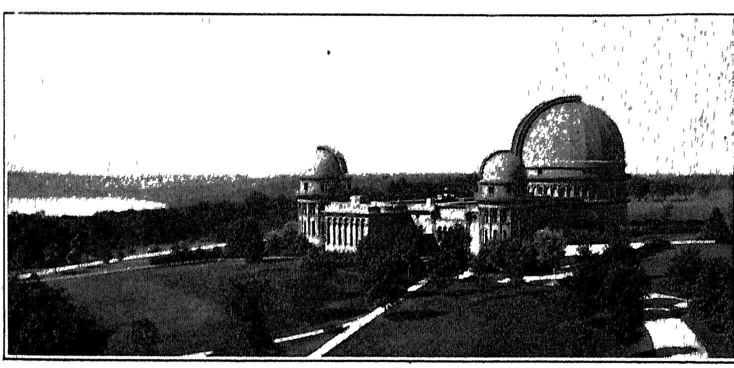
REFERENCES

Reference is made, at the end of each chapter, to a short list of books which contain fuller accounts of the matter treated in the chapter, for current material and the details of specific investigations the student must consult the periodicals and the observatory publications. The most important periodicals, and the abbreviations of their titles usually employed, are:

The Astronomical Journal (*A. J.*), dealing mainly with the astronomy of position.
The Astrophysical Journal (*Ap. J.*), which includes all the Mt. Wilson *Contributions*.
Publications of the Astronomical Society of the Pacific (*A. S. P.*).
Popular Astronomy (*P. A.*), largely for the amateur.
Monthly Notices of the Royal Astronomical Society (*M. N.*), containing the work of English astronomers in all branches of the science.
Astronomische Nachrichten (*A. N.*), the principal astronomical periodical on the continent of Europe.
Vierteljahrsschrift der Astronomischen Gesellschaft (*V. J. S*), which contains observatory reports, and statistics of planets, comets, and variable stars.
Bulletin of the Astronomical Institutes of the Netherlands (*B. A. N.*), in English, published jointly by the Dutch observatories.

Summaries of the work done at most observatories appear as articles in the periodicals, but longer accounts, and in some cases short communications as well, appear in observatory publications A few of the most important are:

Harvard Observatory: Annals, Circulars, and Bulletins.
Lick Observatory: Publications and Bulletins.
Dominion Astrophysical Observatory: Publications.
Potsdam Astrophysikalisches Observatorium: Publikationen.



Yerkes Observatory

11. The apparent place of a heavenly body is simply the point where a line drawn from the observer through the body in question, and continued onward, pierces the celestial sphere. It depends solely upon the direction of the body and has nothing to do with its distance (Fig. 1).

The *apparent distance* between two stars is therefore simply a difference in direction, and the *apparent diameter* of the moon is the angular separation between lines of sight to diametrically opposite points of the moon's disk (Fig. 42). Obviously, angular units alone can properly be used in describing apparent distances in the sky. One cannot say correctly that the two stars known as the pointers are so many *feet* apart; their distance is approximately five *degrees*, — which is the length of the arc of the great circle on the celestial sphere connecting the two stars.

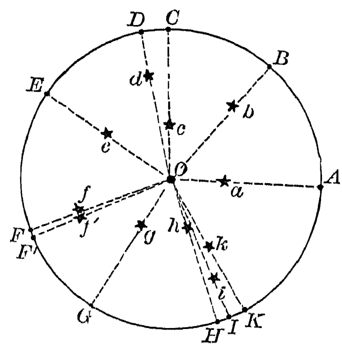


FIG. 1. Apparent Places of Stars on the Celestial Sphere

A, *B*, *C*, etc are the apparent places of *a*, *b*, *c*, etc , the observer being at *O* Objects that are nearly in line with each other, such as *h*, *i*, *k*, will appear close together in the sky, however great the real distance between them The moon, for instance, often looks to us very near a star, which is always a great distance beyond

The student of astronomy should accustom himself as soon as possible to estimating celestial measures in angular units. A little practice soon makes it easy, although the beginner is likely to be embarrassed by the fact that the sky appears not as a true hemisphere but as a flattened vault, so that all estimates of angular distances for objects near the horizon are likely to be exaggerated. The moon when rising or setting looks to most persons much larger than when overhead, and the "Dipper bowl" when underneath the pole seems to cover a much larger area than when above it.

These illusions are directly traceable to the unconscious habit, developed from an early age, of interpreting apparent size by the aid of our familiarity with real size. This perspective adjustment is strongly developed in the horizontal plane, to which our experiences are largely confined, but is very imperfect in the vertical direction. It is worth remarking that a ship seen below an airplane looks much smaller than one seen on the horizon at the same distance.

to the direction of gravity, the horizon may also be defined as the great circle in which a plane tangent to a surface of still water at the place of observation cuts the celestial sphere.

The word "horizon" (from the Greek) means literally "the boundary," that is, the limit of the landscape, where sky meets earth or sea. This boundary line is known in astronomy as the *visible horizon.* On land it is irregular, but at sea it is practically a true circle, nearly coinciding with the horizon above defined.

14. Vertical Circles; the Meridian and the Prime Vertical. *Vertical circles* are great circles drawn from the zenith at right angles to the horizon, and therefore passing through the nadir also. Any point in the heavens has one of these circles passing through it.

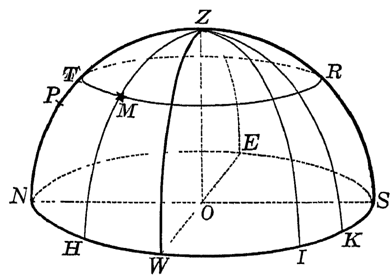


FIG. 2 The Horizon and Vertical Circles

O, the place of the observer, OZ, the observer's vertical, Z, the zenith, P, the pole, $SWNE$, the horizon, $SZPN$, the meridian; EZW, the prime vertical, M, some star; ZMH, arc of the star's vertical circle, TMR, the star's almucantar, angle SZM, or arc SWH, star's azimuth; arc HM, star's altitude; arc ZM, star's zenith distance

That particular vertical circle which passes north and south, through the pole (to be defined hereafter), is known as the *celestial meridian* and is the circle traced upon the celestial sphere by the plane of the terrestrial meridian upon which the observer is located. The vertical circle at right angles to the meridian is called the *prime vertical.* The points where the meridian intersects the horizon are the *north* and *south points*; and the *east* and *west points* are midway between them. These are known as the four *cardinal points.*

The *parallels of altitude*, or *almucantars*, are small circles of the celestial sphere parallel to the horizon, sometimes called *circles of equal altitude.*

15. Altitude and Zenith Distance. The *altitude* of a heavenly body is its angular elevation above the horizon, that is, the number of degrees between it and the horizon, measured on the vertical circle passing through the object. Referring to Fig. 2, the vertical circle ZMH passes through the body M.

of five stars, each about as bright as the polestar itself. This is the constellation of Cassiopeia.

If now we watch these stars for only a few hours, we shall find that while all the configurations remain unaltered, their places in the sky are slowly changing. The Dipper slides downward toward the north, so that by eleven o'clock the pointers are

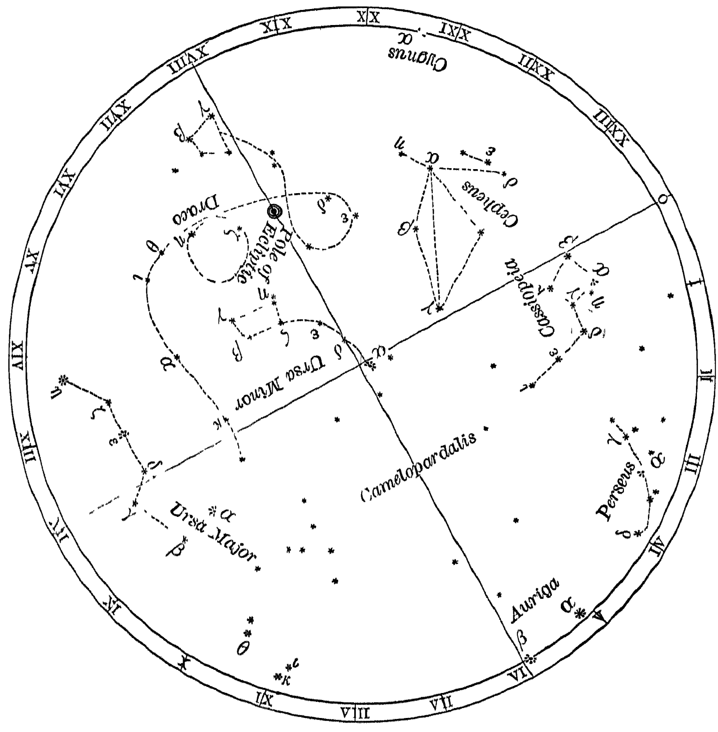


FIG. 3. The Northern Circumpolar Constellations

directly under Polaris. Cassiopeia still keeps opposite, however, rising toward the zenith; and if we were to continue to watch them all night, we should find that all the stars appear to be moving in circles around a point near the polestar, revolving in the opposite direction to the hands of a watch (as we look up toward the north) with a steady motion which takes them completely around once a day, or, to be exact, once in the *sidereal day*, which consists of $23^h\ 56^m\ 4^s.1$ of ordinary time. Thus the

ing the center of the small diurnal circle described by some star near it, as, for instance, the polestar.

Since the two poles are diametrically opposite in the sky, only one of them is usually visible from a given place; observers north of the equator see only the north pole, and vice versa in the southern hemisphere.

Knowing as we now do that the apparent revolution of the celestial sphere is due to the real rotation of the earth on its axis, we may also define the poles as the *two points where the earth's axis of rotation* (*or any set of lines parallel to it*), *produced indefinitely, would pierce the celestial sphere.*

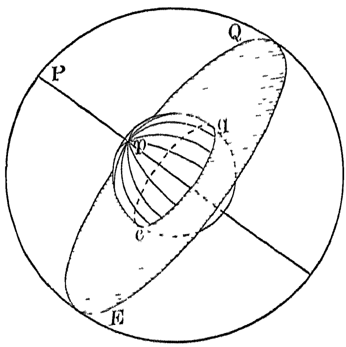


FIG. 5. The Plane of the Earth's Equator

When produced to cut the celestial sphere, this plane traces out the celestial equator

19. The Celestial Equator, and Hour-Circles. *The celestial equator is the great circle of the celestial sphere, drawn halfway between the poles* (and therefore everywhere 90° from each of them), *and is the great circle in which the plane of the earth's equator cuts the celestial sphere,* as illustrated in Fig. 5. Small circles drawn parallel to the celestial equator, like the parallels of latitude on the earth, are called *parallels of declination.* A star's parallel of declination is identical with its *diurnal circle.*

The great circles of the celestial sphere, which pass through the poles in the same way as the meridians on the earth, and which are therefore perpendicular to the celestial equator, are called *hour-circles.* Each star has its own hour-circle, which apparently moves with it. *That particular hour-circle which at any moment passes through the zenith of the observer coincides with the celestial meridian,* already defined.

20. Declination and Hour Angle. *The declination of a star is its distance in degrees north or south of the celestial equator,* + *if north and* — *if south.* It corresponds closely to the *latitude* of a place on the earth's surface, but cannot be called *celestial latitude,* because the term has been preëmpted for an entirely different

The ecliptic cuts the celestial equator in two opposite points at an angle of about $23\frac{1}{2}°$. These points are the *equinoxes.* The *vernal equinox,* or *first of Aries* (symbol ♈), *is the point where the sun crosses from the south to the north side of the equator,* on or about the twenty-first of March. The other is the *autumnal equinox.*

The angle at which the ecliptic and equator intersect is called the *obliquity of the ecliptic.*

The summer and winter *solstices* are points on the ecliptic, midway between the two equinoxes and 90° from each, where the sun attains its extreme declination of $+23\frac{1}{2}°$ and $-23\frac{1}{2}°$ — in summer and winter respectively, in the northern hemisphere.

The hour-circles drawn from the celestial pole through the equinoxes and solstices are called the *equinoctial* and the *solstitial colure* respectively.

Neglecting for the present the gradual effect of precession (§ 166), these points and circles are fixed with reference to the stars, and form a framework by which the places of celestial objects may be conveniently defined and catalogued.

No conspicuous star marks the position of the vernal equinox, but a line drawn from the polestar through β Cassiopeiæ and α Andromedæ, and continued another 30°, will strike very near it.

22. Right Ascension. The *right ascension* of a star may now be defined as *the angle made at the celestial pole between the hour-circle of the star and the hour-circle which passes through the vernal equinox* (called the *equinoctial colure*), or as *the arc of the celestial equator intercepted between the vernal equinox and the point where the star's hour-circle cuts the equator.* Right ascension is reckoned always *eastward* from the equinox, completely around the circle, and may be expressed either in degrees or in time units. A star one degree *west* of the equinox has a right ascension of 359°, or $23^h\,56^m$.

Evidently the diurnal motion does not affect the right ascension of a star, but, like the declination, it remains practically unchanged for years.

The symbols are for the most part conventional pictures of the objects. The symbol for Aquarius is the Egyptian character for water. The origin of the signs for Leo, Capricornus, and Virgo is not clear.

25. Galactic Coördinates. One more system of coordinates — a very modern one — deserves mention. In this the *plane of the Milky Way* is taken as fundamental, and latitudes are measured from the *galactic equator*, which is its trace on the sky, while longitudes are reckoned along this circle, starting from its intersection with the celestial equator in $18^h\,40^m$ right ascension. These *galactic coördinates* are used in studies of the apparent distribution of the stars in space. The concentration of the stars toward this plane renders it far more fundamental than any of the others with which we have dealt; but since the Milky Way is a broad and ill-defined belt in the sky, the position of its central line cannot be precisely determined by observation, and galactic coördinates are therefore not suitable for the precise specification of the positions of the heavenly bodies.

The accepted position of the north galactic pole is $12^h\,40^m$ right ascension and 28° north declination, so that the galactic equator cuts the celestial equator at an angle of 62°.

26. Recapitulation. The *direction of gravity* at the point where the observer happens to stand determines his *zenith*, *nadir*, and *horizon*, the *almucantars*, or *parallels of altitude*, and all the *vertical circles*. One of the vertical circles, the *meridian*, is singled out from the rest by the circumstance that it *passes through the pole*, marking the *north* and *south points* where it cuts the horizon. *Altitude* and *azimuth* define the position of a body by reference to the horizon and meridian.

This set of points and circles shifts its position among the stars with every change in the place of the observer and with every moment of time.

In a similar way the *direction of the earth's axis*, which is independent of the observer's place on the earth, determines the *celestial pole*, the *equator*, the *parallels of declination*, and the *hour-circles*. Two of these hour-circles are singled out as reference lines. One of them is the hour-circle which at any moment passes through the zenith and coincides with the meridian, — a

27. The scheme given on page 20 presents in tabular form the relations of the five different systems to each other. In each case one of the two coördinates is measured along a *primary* great circle, from a point selected as the *origin* to a point where a *secondary* circle cuts it, drawn through the object perpendicular to the primary. The second coordinate is the angular distance of the object from the primary circle measured along this secondary.

Still other coördinate systems are possible. For example, a planet has an equator and equinox of its own, to which the positions of the heavenly bodies would naturally be referred by an observer on its surface. *Planetocentric* and *selenographic* coordinates of the earth and sun are of use to observers of the surface markings of Mars, Jupiter, and the moon, and are given in the *American Ephemeris*.

LATITUDE, LONGITUDE, AND TIME

28. Astronomical Latitude. Evidently the appearance of the heavens for any observer will be radically influenced by the distance between his zenith and the equator. This quantity is called the *astronomical latitude* and is defined as the angle between the observer's *vertical* and the plane of the equator. This is obviously equal to the *declination of the zenith* and, almost as obviously, to the *altitude of the pole*; for PN and ZQ (Fig. 7) may be obtained by subtracting PZ from PQ or ZN, each of which equals 90°. These fundamental relations cannot be too strongly emphasized.

Since the earth is not exactly round, the distance of a point from the earth's equator measured on the earth's surface is not an exact measure of the astronomical latitude (Fig. 8).

29. The Right Sphere. If the observer is situated at the earth's equator, that is, in latitude *zero*, the pole will be in his horizon, and the celestial equator will be a vertical circle, coinciding with the prime vertical (§ 14). All heavenly bodies will *rise and set vertically*, and their diurnal circles will all be bisected by the horizon, so that they will be twelve hours above and twelve hours below it; and the length of the night will always equal that of the day (neglecting refraction, § 114). This aspect of the heavens is called the *right sphere*.

Since the sun and the moon move among the stars in such a way that half the time they are north of the equator and half the time south of it, they will be half the time above the horizon and half the time below it (again neglecting refraction). The moon will be visible for about a fortnight each month, and the sun for about six months each year.

It is worth noting that for an observer *exactly* at the north pole the definitions of meridian and azimuth break down, since at that point the zenith coincides with the pole. Facing in whichever direction he will, he is still looking directly *south*. If he changes his place a few steps, however, his zenith will move, and everything will become definite again.

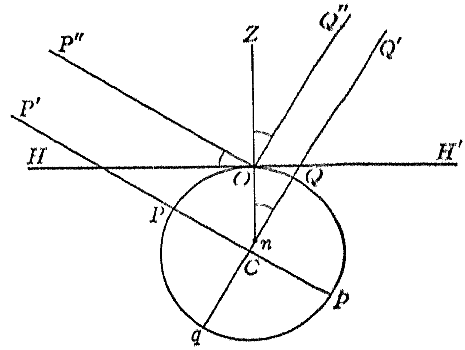


FIG. 8. Altitude of the Pole Equals the Observer's Latitude

This is a section of the earth through the observer's meridian. The observer is at O. His zenith is in the direction of Z, and his horizon is represented by HOH'. P'' lies in the direction of the pole, and the line OQ'' is in the plane of the celestial equator. The altitude of the pole, HOP'', equals the declination of the zenith, $Q''OZ$, which equals QnO, the astronomical latitude of the observer — On representing the direction of the nadir

31. The Oblique Sphere. At any station between the poles and the equator the pole will be elevated above the horizon, and the stars will rise and set in *oblique circles*, as shown in Fig. 9. Those whose distance from the elevated pole is less than PN (the latitude of the observer) will of course never set, remaining perpetually visible. The radius of this *circle of perpetual apparition*, as it is called (the shaded cap around P in the figure), is obviously just equal to the height of the pole, becoming larger as the latitude increases. On the other hand, stars within the same distance of the depressed pole will lie in the *circle of perpetual occultation* and will never rise above the horizon. A star exactly on the celestial equator will have its diurnal circle bisected by the horizon and will be above the horizon twelve hours. A star north of the equator, if the north pole is the elevated one, will have more than half its diurnal circle above the horizon and will be visible

In astronomy three kinds of time are now recognized, *sidereal time*, *apparent solar time*, and *mean solar time*, the last being essentially the time of civil life and ordinary business, while the first is used exclusively for astronomical purposes.

34. Sidereal Time. The celestial object which determines sidereal time, by its position in the sky at any moment, is the *vernal equinox*, or *first of Aries*.

The local sidereal *day* begins at the moment when the first of Aries crosses the observer's meridian, and the *sidereal time* at any moment *is the hour angle of the vernal equinox*. It would be marked by a perfect clock so set and adjusted as to show *sidereal noon* ($0^h\ 0^m\ 0^s$) at each transit of the first of Aries.

The equinoctial point is, of course, invisible; but its position among the stars is always known, so that its hour angle at any moment can be determined by observing the stars. On account of the precession of the equinoxes (§ 166) the sidereal day thus defined is slightly shorter than it would be if defined by the transits of a point *fixed among the stars*. This difference amounts, on the average, to 1/120 of a second; but as the motion of the equinox is not uniform, not all sidereal days are of exactly the same length. The differences between them, however, are much smaller than the errors of the best clocks, and may be neglected.

35. Apparent Solar Time. Just as sidereal time is the hour angle of the vernal equinox, so *apparent solar time* at any moment is *the hour angle of the sun*. It is the *time shown by the sundial*, and its noon occurs at the moment when the sun's center crosses the meridian.

On account of the earth's orbital motion the sun appears to move eastward along the ecliptic, completing its circuit in a year. Each noon, therefore, it occupies a place among the stars about a degree farther east than it did the noon before, and so comes to the meridian about four minutes *later*, if time is reckoned by a *sidereal clock*. In other words, the solar day is about four minutes *longer* than the sidereal, the difference amounting to *exactly one day* each year, which contains $366\frac{1}{4}$ sidereal days.

But the sun's eastward motion is, for several reasons, not uniform, and the apparent solar days vary in length (§ 170). December 23, for instance, is about fifty-one seconds longer from

midnight and reckoned around through the whole twenty-four hours. Before this date an *astronomical* day was employed, which began twelve hours earlier. The reason was that astronomers are "night birds" and found it inconvenient to change dates at midnight, in the middle of their work. This must be borne in mind in referring to almanacs of 1924 or earlier; thus, 10 o'clock (10 A.M.) of Wednesday, February 27, *civil* reckoning, is Tuesday, February 26, 22 o'clock, by the old *astronomical* reckoning. *Civil time* is therefore obtained by *adding twelve hours to mean solar time* (which may change the day, the month, or even the year).

This is the present usage (1926) of the American and French nautical almanacs. The *British Nautical Almanac* of 1926 uses the designation "mean time" for what is here called "civil time." The French call Greenwich civil time *temps universel* or *temps civil de Greenwich*; the Germans, *Weltzeit*. It is greatly to be hoped that an international agreement may soon be reached. Meanwhile the meaning of the terms employed in each nautical almanac is clearly explained in the text.

38. Terrestrial Longitude. The longitude of a place on the earth may be defined as *the angle at the celestial pole between a standard meridian and the meridian of the place, or as the arc of the celestial equator intercepted between the two meridians.*

It would be equally permissible to measure this angle at the pole of the earth if it were not for the effects of deviations of the vertical (§ 142).

There is no inherent reason why one meridian should be chosen as a standard rather than another, and the decision must be arbitrary. The *meridian of Greenwich*, by international agreement, is now used in almost all cases.

This usage arose because Greenwich is the national observatory of England, and British ships naturally referred their longitudes to its meridian. The system was thus spread all over the world, and other countries have gradually adopted it, the advantages of having a single system overcoming local prejudices. From the definition of hour angle it follows that the difference between the hour angles of any celestial object, as seen from two places at the same instant, is equal to the difference of their longitudes, and therefore that *the longitude of any observer is*

40. Where the Day Begins. If we imagine a traveler starting from Greenwich on Monday noon and journeying westward as swiftly as the earth turns to the east under his feet, he would, of course, keep the sun exactly on the meridian all day long and have continual noon. But what noon? It was Monday when he started, and when he gets back to London, twenty-four hours later, it is Tuesday noon there, although he has seen no intervening sunset. When does Monday noon become Tuesday noon? It is agreed among mariners *to make the change of date at the 180th meridian from Greenwich*, which passes over the Pacific, hardly anywhere touching land.

Ships crossing the line *from the east skip one day* in so doing. If it is Monday when a ship reaches the line, it becomes Tuesday when she passes it, the intervening twenty-four hours being dropped from the reckoning on the log-book. Vice versa, when a vessel crosses the line from the *western* side it counts the same day *twice over*, passing from Tuesday back to Monday and having to do Tuesday over again.

There is considerable irregularity in the date actually used on the different islands in the Pacific, as will be seen by looking at the so-called *date-line* as given in the Century Atlas of the World. Those islands which received their earliest European inhabitants via the Cape of Good Hope have adopted the Asiatic date, even if they really lie east of the 180th meridian, while those that were first approached from the American side have the American date. When Alaska was transferred from Russia to the United States, it was necessary to drop one day of the week from the official dates.

TRANSFORMATION OF COÖRDINATES

The almanac gives the right ascension and declination of the heavenly bodies. To point a telescope at a faint star we must have either the altitude and the azimuth, or the hour angle and the declination. The transformation in the latter case is very simple.

41. Hour Angle, Right Ascension, and Sidereal Time. From Fig. 7 and the definition of the quantities, taking into account the direction in which they are measured, we see that the *hour angle of any point may be obtained by subtracting its right ascension from the sidereal time*; that is, $t = \theta - \alpha$ (another relation of fundamental

43. Conversion of Time. It is very often necessary to convert the time at one place into the corresponding time at another, or the time measured in one system into that measured in a different one.

To find the time at one place when that at another is given, it is only necessary to *add or subtract the difference of longitude*, remembering that the time at the eastern station is always fast, compared with that at the western, unless the date-line intervenes. It makes no difference what kind of time is concerned, — sidereal, apparent, or mean, — so long as it is *measured in the same system at both stations* (§ 38).

A special case of this is the conversion of local mean time into standard time, which is accomplished by adding or subtracting the difference between the observer's longitude and that of the "standard time meridian." (This amounts to about -4^m at New York, the local mean time being 4^m faster than Eastern standard.)

To convert *apparent solar time into mean solar time* it is necessary to apply the *equation of time*, adding or subtracting as the case may be. At a given instant this correction is the same for all places, but it varies from day to day. Its exact value at the beginning of each Greenwich civil day is given in the *Nautical Almanac*, and the value at any other time may be found by interpolation. An approximate value of the equation of time (within half a minute or so) may be taken from Fig. 65 (p. 147); 12 hours must be added to the mean time to obtain the civil time.

44. Sidereal and Mean Time Intervals. The time divisions on the two systems are not of the same length. Since the tropical year (§ 175) contains 365.2422 mean solar days, and exactly one more sidereal day, it follows that the number of *sidereal* seconds in any time interval is equal to the number of *mean solar* seconds multiplied by $\frac{366.2422}{365.2422}$, that is, by 1.00273791. Hence, if I and I' are respectively the number of mean solar and sidereal seconds in any time interval, we have $I' = I + 0.00273791\, I$.

Conversely,

$$I = I' \times \frac{365.2422}{366.2422} = I' \times 0.99726957 = I' - 0.00273043\, I'.$$

price. They contain *ephemerides* giving, besides other data, the right ascension and declination of the sun, moon, and planets at regular intervals of time, and also of a large number of "clock stars," which are observed for the determination of time. They also contain predictions of eclipses, occultations, and other phenomena. The work of computation (which is very heavy) is divided among the various offices by international agreement. The publication of those data which are of use only to astronomers is also divided among the various almanacs,[1] so that all are needed at an observatory.

A celestial globe will be of great assistance in studying the diurnal phenomena. By means of a globe it can be seen at once which stars never set, which ones never rise, and during what part of the twenty-four hours a heavenly body at a known declination is above or below the horizon.

47. The Celestial Globe. The celestial globe is a ball, usually of papier-mâché, upon which are drawn the circles of the celestial sphere and a map of the stars. It is mounted in a framework which represents the horizon and the meridian, in the manner shown in Fig 11.

The *horizon*, HH' in the figure, is usually a wooden ring three or four inches wide, directly supported by the pedestal. It carries upon its upper surface, at the inner edge, a circle marked with degrees for measuring the azimuth of any heavenly body, and outside this the so-called zodiacal circles, which give the sun's longitude and the equation of time (§§ 36 and 169) for every day of the year.

The *meridian ring*, MM', is a circular ring of metal which carries the bearings of the axis on which the globe revolves. Things are, or ought to be, so arranged that the mathematical axis of the globe is exactly in the same plane as the graduated face of the ring, which is divided into degrees and fractions of a degree, with zero at the equator The meridian ring fits into two notches in the horizon circle and is held underneath the globe by a support with a clamp, which enables us to fix it securely in any desired position, the mathematical center of the globe being precisely in the plane both of the meridian ring and of the horizon.

The *hour index* on the globe here figured is a pointer like the hour-hand of a clock, so attached to the meridian ring at the pole that it can be turned around the axis with stiffish friction, but will retain its position unchanged

[1] The most important are the *American Ephemeris and Nautical Almanac* (Superintendent of Documents, Government Printing Office, Washington, price $1.00), the *Nautical Almanac* (London), *Connaissance des Temps* (Paris), and *Berliner Jahrbuch.*

The positions of the moon and planets are not given by this operation, since they have no fixed places in the sky and therefore cannot be put upon the globe by the maker. If one wants them represented, he must look up their right ascensions and declinations for the day in some almanac and mark the places on the globe with bits of wax or paper.

All the problems involving transformation of coordinates may be solved very rapidly and with considerable accuracy by measurement on a celestial globe. It is only necessary to cut a strip of stiff paper and graduate this along one edge to correspond with the degrees on the equator of the globe. The distance in degrees between any two celestial objects may then be measured with this strip. By placing one end of it at the point representing the zenith, and carrying it down past any object to the horizon, the altitude of the object may be read off on the strip, and its azimuth on the horizon. Such measures, on an ordinary globe, are liable to errors of a degree or so.

EXERCISES

1. What point in the celestial sphere has both its right ascension and its declination zero? What are the celestial latitude and longitude of this point?

2. What are the hour angle and azimuth of the zenith?

3. At what points does the celestial equator cut the horizon?

4. What angle does the celestial equator make with the horizon at these points, as seen by an observer in latitude 40°? What if his latitude is 10°? 20°? 50°? 60°?

5. When the vernal equinox is rising on the eastern horizon, what angle does the ecliptic make with the horizon at that point for an observer in latitude 40°? what angle when it is setting?

6. What are the approximate right ascension and declination (α and δ) of the sun on March 21 and September 22?

7. On March 21, one hour after sunset, what would be the position of a star having a right ascension of seven hours and a declination of 40°, the observer being in latitude 40°?

8. If a star rises tonight at 10 o'clock, at what time (approximately) will it rise 30 days hence?

9. What are the longitude (λ) and latitude (β) of the sun when its right ascension is six hours? When its α is twelve hours?

10. What are the latitude and longitude of the north celestial pole?

11. Under what circumstances will the pole of the ecliptic be at an observer's zenith? Show that in this case $h = \beta$ and $A = 270 - \lambda$.

12. How much will a sidereal clock gain on a mean solar clock in 10 hours and 30 minutes? *Ans.* $1^m\ 43^s\ 5$.

CHAPTER II

ASTRONOMICAL INSTRUMENTS

TELESCOPES AND THEIR ACCESSORIES AND MOUNTINGS · TIMEKEEPERS AND CHRONOGRAPHS · THE TRANSIT INSTRUMENT · THE MERIDIAN CIRCLE · THE MICROMETER · MEASUREMENT OF PHOTOGRAPHS · THE SEXTANT

Astronomical observations are of various kinds — the surface of a heavenly body is to be examined minutely; the position which it occupies at a given time, or the time at which it arrives at a given circle of the sky (usually the meridian), is to be ascertained; its brightness is to be measured or its spectrum investigated; a region of the sky is to be photographed and the relative positions of a number of stars determined from measurement of the plate. The telescope has been adapted to many purposes, and accessories have been devised for special problems.

49. The Telescope. The most important of all astronomical instruments is the telescope. Its principal uses are of three kinds: (1) to form an image of a heavenly body which may be observed with high magnifying power, measured with precision, or photographed; (2) to collect the light from a heavenly body and feed it into some other instrument, such as a photometer or spectroscope, with which the brightness, color, or composition of the light may be studied; and (3) to act as a pointer in making precise observations of the direction of a star.

Telescopes are of two kinds, refracting and reflecting. The former were invented first and are in more general use, but the largest instruments ever made are reflectors. The fundamental principle is the same in both: the large lens, or mirror, of the instrument forms at its focus a *real image* of the object looked at, and this image is then examined and magnified by the eyepiece, which in principle is only a magnifying-glass.

In the form of telescope introduced by Galileo,[1] however, and still used as the opera-glass, the rays from the object-glass are

[1] See Grant's *History of Astronomy*, p 514 ff , on the invention of the telescope.

its image subtend equal angles, since rays that pass through the point *o* suffer no sensible deviation.

If the focal length of the lens A is 10 feet, then the image of the moon formed by it will appear, when viewed from a distance of 10 feet, just as large as the moon itself, from a distance of 1 foot the image will, of course, appear ten times as large.

With such an object-glass, therefore, one can see the mountains of the moon and the satellites of Jupiter by removing the eyepiece and putting the eye in the line of the rays, at a distance of 10 or 12 inches from the eyepiece tube.

51. Magnifying Power. With the naked eye one cannot, unless near-sighted, see the image distinctly from a distance much less than 10 inches; but if a magnifying-lens of 1-inch focus is used, the image can be viewed from a distance of only an inch, and it will look ten times larger in diameter. The *magnifying power* is equal to the *quotient obtained by dividing the focal length of the object-glass by that of the eye-lens*, or, as a formula, $M = F/f$. The magnifying power of the telescope is changed at pleasure by simply changing the eyepiece (§ 59).

If, for example, the focal length of the object-glass be 4 feet and that of the eye-lens $\frac{1}{4}$ inch, then

$$M = 48 \div \tfrac{1}{4} = 4 \times 48 = 192$$

52. Brightness of Image. This depends not upon the focal length of the object-glass but upon its size, — its area. If we estimate the diameter of the pupil of the eye at one fifth of an inch, then (neglecting the loss in transmission through the lenses) a telescope 1 inch in diameter collects into the image of a star 25 times as much light as the naked eye receives; and the great Yerkes telescope, 40 inches in diameter, gathers 40,000 times as much, or about 35,000 after allowing for the losses.

With a large telescope, therefore, objects like the stars, which appear as mere luminous points, have their brightness immensely increased, and millions otherwise invisible are brought to view. To obtain the full benefit of the aperture the entire pencil of light emerging from the eyepiece must be small enough to enter the pupil of the eye. It is clear from Fig. 12 that the ratio of the diameter of the object-glass to the diameter of the *emergent pencil* equals that of the focal lengths of the two lenses, that is,

really gained by pushing the magnifying power beyond the point at which the diffuseness of the image, arising from this cause, becomes conspicuous, that is, beyond a power of from 50 to 60 to the inch of aperture. For most purposes lower powers are more satisfactory; indeed, it is a good practical rule not to use a higher magnifying power than suffices to show the desired details clearly.

This effect of diffraction has much to do with the superiority of large instruments in showing minute details and in separating close double stars (Fig. 13). The full theoretical *resolving power* of a telescope is only practically available when the air is perfectly steady (which, unfortunately, seldom happens in most

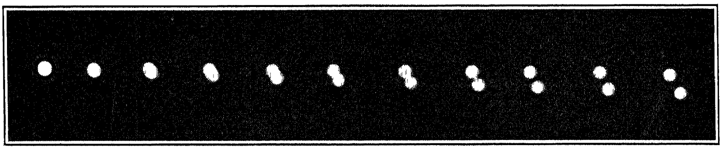


FIG 13 Resolving Power

The diffraction image of an artificial double star has been observed through a telescope of small aperture. The first image (from the left) shows the diffraction disk of a single star with the first two rings surrounding it, the others those of double stars with steadily increasing separation. Duplicity is evident in the third and fourth images, but clear separation of the diffraction disks occurs first in the ninth The actual telescopic image of a star is very rarely as sharp as this (From a photograph by J A Anderson)

climates). A given amount of atmospheric disturbance (§ 118) injures the performance of a large telescope much more than that of a small one; on bad nights the performance may actually be improved by reducing the aperture by means of a diaphragm in front of the objective.

54. Imperfections of Lenses; Spherical and Chromatic Aberration. A single lens, with spherical surfaces, cannot even approach this ideal performance, but suffers necessarily from various *aberrations*, of which the principal are the *spherical* and the *chromatic*. The former consists in a difference of the focal length for rays which have passed through the lens near its center and near the edge; the latter (which is more serious), in a difference in the focal length for light of different colors (wave-lengths).

The blue and violet rays are more refrangible than the yellow and red, and are brought to a focus nearer the objective, so that

with Canada balsam or some other transparent medium. At present some of the best makers separate the two lenses by a considerable distance, so as to admit a free circulation of air between them.

56. Secondary Spectrum. It is not possible, however, with the kinds of glass ordinarily employed, to secure a perfect correction of the color. The best achromatic lenses bring the yellowish-green rays to a focus nearer the lens than they do the red and violet. In consequence the image of a bright star is surrounded by a purple halo, which is not very noticeable in a good telescope of small size but is very conspicuous and troublesome in a large instrument, and makes it difficult to see faint stars close to a bright one.

By using the new varieties of glass which are now made at Jena, objectives of three components can be constructed which are very nearly *aplanatic*, that is, sensibly free from both chromatic and spherical aberration. Several telescopes of this sort, of apertures up to 12 inches, are in use and perform admirably; but it has not yet been possible to get disks of these glasses large enough for lenses of great size.

57. Photographic Telescopes. A visually corrected objective is practically useless for astronomical photography on ordinary plates, for the blue and violet rays, which are photographically the most effective, are not brought to any one sharp focus. When the photographic light of but a single star or planet is needed, it may be brought to one focus by introducing a correcting lens, much smaller than the objective, a few feet above the eye end. Excellent photographs of larger fields may be made, without any correcting lens, by using isochromatic plates and placing just in front of the plate a color screen (a plane-parallel glass coated with a thin film of gelatin stained with a yellow dye, which absorbs the blue and violet light); but the necessary exposures are relatively long.

When a telescope is to be used primarily for photography, the objective is designed, in the first place, so as to bring all the blue and violet rays to substantially the same focus, leaving the yellow and red uncompensated. Such a telescope is almost useless visually, but permits very much shorter exposures than the devices described above.

used in connection with such a reticle. In order to make the lines of the reticle visible at night a faint light is reflected into the instrument by some one of various arrangements devised for the purpose.

61. The Reflecting Telescope. About 1670, when the chromatic aberration of refractors first came to be understood (in consequence of Newton's discovery of the decomposition of light), the reflecting telescope was invented. For nearly one hundred and fifty years it held its place as the chief instrument

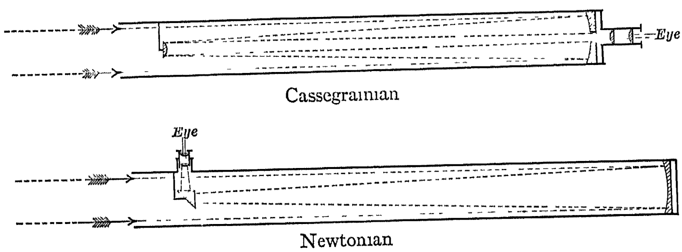


FIG. 15. Reflecting Telescopes

A concave mirror reflects the rays of light to a focus. In the Newtonian form the light is brought, for convenience, by plane reflection to the side of the tube The main characteristic of the Cassegrainian is the reflection on a secondary convex mirror with the consequent increase of effective focal length

for star-gazing; it was almost displaced by the refractor during the nineteenth century but returned to importance in the twentieth.

The essential part of a reflector is the great mirror, which brings the light of the stars to a focus. The direct image so produced is in a very inconvenient position, and various means are adopted to bring it to a point where it can be utilized.

In the *Newtonian* form (Fig. 15) a small plane mirror standing at an angle of 45° intercepts the rays a little before they come to their focus, and throws them to the side of the tube, where the eyepiece is placed.

In the *Cassegrainian* form a small convex mirror, placed in about the same position, reflects the rays back through a hole in the center of the large mirror. With this form the observer looks directly at the stars, as with a refractor, and the image is erect.

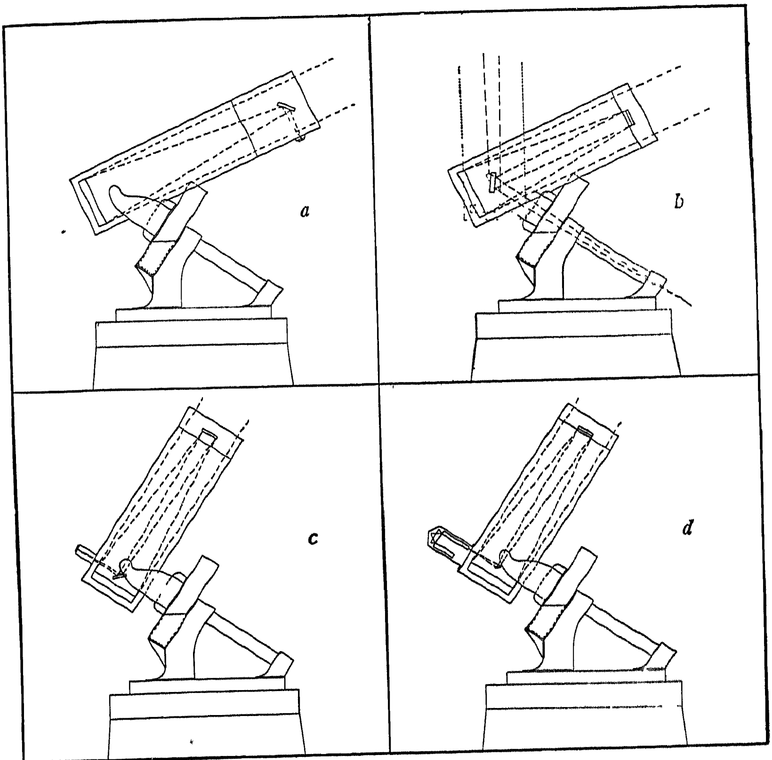


FIG 16. Four Arrangements of the 60-Inch Reflector of the Mt. Wilson Observatory

The focal length of the 60-inch mirror is 25 feet. In the Newtonian form, *a*, the small secondary mirror is flat and simply reflects the light to the side of the tube. The focal length remains 25 feet For use as a Cassegrain reflector the upper section of the tube is replaced by a shorter section carrying a convex mirror, which increases the equivalent focal length. By the use of a plane mirror the light is brought to the side of the tube. Different focal lengths can be obtained by the use of different convex mirrors. In the arrangement *d*, where the light is brought to a spectroscope at the side of the tube, the focal length is 80 feet; in *c*, which is used for large-scale photography, it is 100 feet; and in the coudé form, *b*, with a focal length of 150 feet, the light is brought down the polar axis to a large stationary spectroscope

advantages in spectroscopic work. In photography the great reflectors, with their great light power, excel in the observation of faint objects such as nebulæ. The doublet stands alone for work requiring a wide field.

63. Great Telescopes. The largest telescope in the world at present is the 100-inch reflector of the Mt. Wilson Observatory

65. The advantages of the equatorial mounting are very great. In the first place, when the telescope is once pointed upon an object, it is not necessary to turn the declination axis at all in order to keep the object in view, but only to turn the *polar axis* with a perfectly uniform motion, which can be, and usually is, given by a *driving-clock* (Fig. 18). The driving-clock is driven by a heavy weight and controlled by a heavy centrifugal governor, which acts continuously, not by jerks, as in an ordinary clock. An electric motor drive, synchronized by a pendulum clock, has also been used.

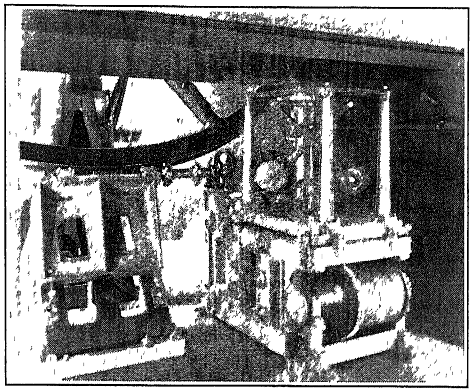


FIG 18. The Driving-Clock of the 100-Inch Hooker Telescope

The power is transmitted through the worm gear to the worm wheel (17 feet in diameter), which may be clamped to the polar axis The centrifugal governor is seen above, at the right

In the next place, it is very easy to find an object, even if invisible to the eye (like a faint comet or a star in the daytime), provided we know its right ascension and declination and have the sidereal time, *a sidereal clock or a chronometer being an indispensable accessory of the equatorial.* Set the declination circle to the declination of the object and then turn the polar axis until the hour circle shows the proper *hour angle,* which is simply the difference between the right ascension of the object and the sidereal time at the moment (§ 41). When the telescope has been so set, the object will be found in the field of view, *provided a low-power eyepiece is used.* On account of refraction the setting does not direct the instrument precisely to the apparent place of the object, but only very near it. Large instruments are provided with *finders* (small auxiliary telescopes with low power and wide field of view). When an object is brought to the center of the field of the finder, it is visible in the main instrument.

with great precision *the position of any object* (such as a comet) *relative to neighboring stars of known position* (§ 100).

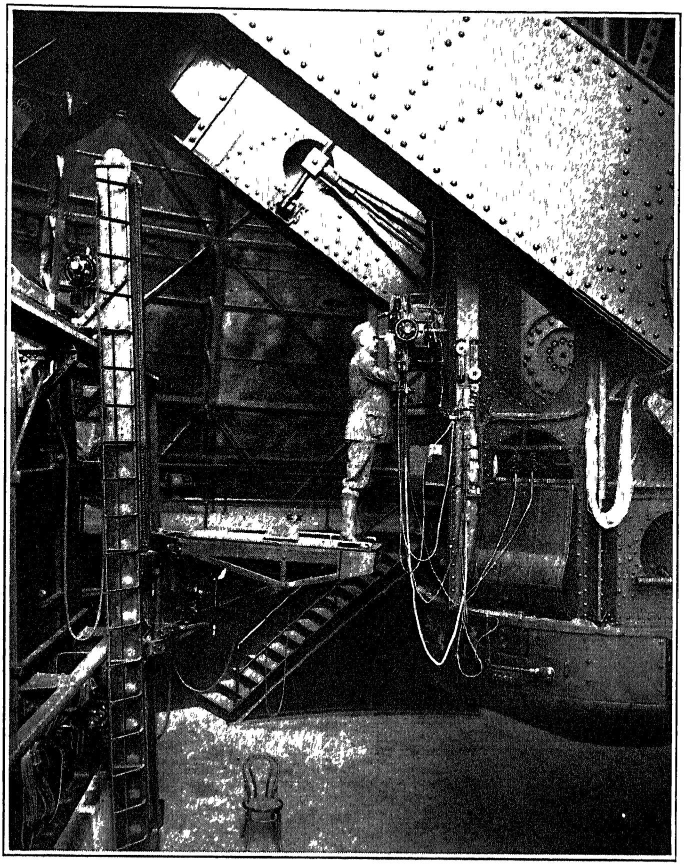


FIG. 20. Observing at the Cassegrain Focus of the 100-Inch Reflector of the Mt. Wilson Observatory

This photograph emphasizes the great size of the telescope. The observer is standing at a height of twelve feet above the floor, on a platform that is movable in any direction by electric motors. The edge of the cell which contains the great mirror may be seen at the bottom of the telescope. A part of the extensive electric wiring system may also be noticed

66. Engineering Details of the Equatorial Mounting. The great modern reflectors are exceedingly heavy (the 100-inch mirror alone weighs 4 tons), and the designing of mountings that will not sag under such loads is a difficult engineering problem. The

the observer, at the eyepiece, to turn the dome as required and to move the instrument in either coördinate, rapidly or slowly. The 100-inch telescope is so rigid that a man may climb out to the upper end witnout shifting a star image seriously in the field of view, and so delicately controlled that it is easy to move the image at will by one third of a second of arc. Such a mounting is a mechanical masterpiece.

As the telescope turns to follow the stars the eyepiece too must move, both horizontally and vertically. With large instruments the resulting inconvenience is minimized by means of a *rising floor*, which may be set at any desired level, or, with the great reflectors, by *observing platforms* movable by electric motors. All this paraphernalia, and the enormous rotating dome (100 feet in diameter for the 100-inch, and 90 feet for the Yerkes 40-inch) add to the already high cost of a great instrument.

The expensive parts of a telescope are the object-glass, the mounting, and the dome. The eyepieces and other accessories are relatively inexpensive. A small telescope, large enough for the purposes of the amateur, can be purchased for a few thousand dollars, or less The 100-inch telescope, with mounting, dome, and accessories, cost $540,000, the largest sum ever spent for any single instrument of research.

67. The Cœlostat. In some cases (especially in work on the sun with instruments of long focus) it is desirable to have the main part of the instrument at rest and to reflect the light into it from a moving mirror.

In the *cœlostat* the plane of the mirror is parallel to that of the polar axis, which carries it directly and revolves only once in forty-eight hours. The reflected image in this case is thrown in a direction determined by the declination of the object. This difficulty can be overcome by using a second mirror to send the reflected beam where it is wanted. With this arrangement the image remains fixed in the focal plane. It is much used for solar work, as in connection with the 150-foot tower telescope of the Mt. Wilson Observatory, for reflecting the beam vertically downward through the objective.

Other forms of mounting, such as the *siderostat* and the *heliostat*, are used for special purposes.

guiding telescope rigidly connected to the photographic telescope. In another arrangement they are placed in a *guiding eyepiece* just outside the plate-holder and moving with it. The guiding is then done by means of screws which move the plate-holder relatively to the rest of the telescope. In this way exposures extending even over several successive clear nights can be made, the plate-holder being shielded from the daylight and the instrument set again, with the aid of the guiding star, upon exactly the same point of the heavens as before.

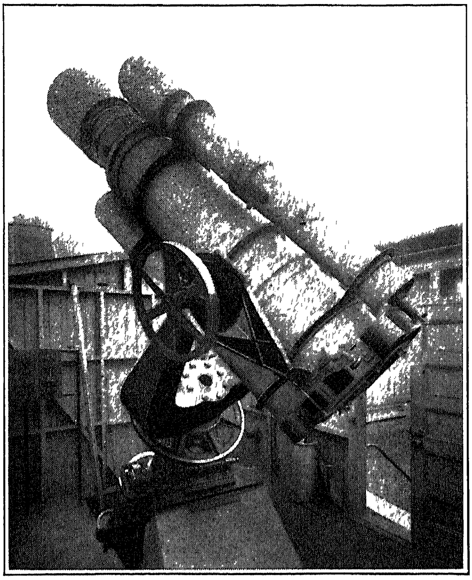


FIG. 23. The Metcalf Telescope of the Harvard College Observatory

The central tube is the 16-inch photographic doublet. The tube flares at the lower end to permit the use of a large plate By exhausting the air in the plate-holder (which is clamped to the lower end of the tube) the plate is given a temporary curvature during the exposure, conforming to the focal surface, which is not plane By this device well-focused star images are secured all over the plate The large ratio of aperture to focal length makes it a very rapid camera · stars of the fifteenth magnitude are photographed in ten minutes The upper tube is the 8-inch guiding telescope; the lower, a photographic telescope of 4-inch aperture. The driving-clock is electrical, and the yoke-mounting avoids the necessity of reversing the instrument at the meridian. (From photograph by Harvard College Observatory)

69. Time-Keepers and Time-Recorders. *The clock, chronometer, and chronograph.* The invention of the pendulum clock by Huygens in 1657 was almost as important to the advancement of astronomy as was that of the telescope by Galileo, and the improvement of the clock and chronometer through the invention of temperature compensation by Harrison and Graham in the eighteenth century is fully comparable to the improvement of the telescope by the achromatic object-glass.

driven by the fall of a weight of only a few grams, which is automatically raised to the top of its run by an electromagnet at intervals of twenty seconds or so, and they will run for months without any attendance.

70. Clock Correction. The *correction* of a clock is the amount that *must be added* to the indication of the clock-face at any moment in order to give the true time; it is therefore plus (+) when the clock is *slow* and minus (−) when it is *fast*. The *rate* of a clock is *the daily change in its correction*, — *plus* (+) when the clock is *losing* and minus (−) when it is gaining.

A perfect clock is one that has a *constant rate*, whether that rate be large or small. It is desirable, for convenience' sake, that both error and rate should be small; but this is a mere matter of adjustment by the user of the clock, who adjusts the error by setting the hands, and the rate by raising or lowering the pendulum bob.

The final adjustment of rate is often obtained by first setting the pendulum bob so that the clock will run slow a second or two daily, and then putting on the top of the bob little weights of a gram or two, which will raise its center of gravity and shorten its period of oscillation. They can be dropped into place or knocked off without stopping the clock or perceptibly disturbing it. A still finer adjustment can be made by altering the pressure within the air-tight clock-case.

71. The Chronometer. Since the pendulum clock is not portable, it is necessary to provide time-keepers that are so. The chronometer is merely a carefully made watch (driven by a coiled spring), with a balance-wheel compensated to run, as nearly as possible, at the same rate in different temperatures, and with a peculiar escapement which, though unsuited to ordinary usage, gives better results than any other when treated carefully. (*Never turn the hands of a chronometer backward; it may ruin the escapement.*)

The box chronometer used on shipboard is usually about five inches in diameter, and is mounted in gimbals so as to remain horizontal at all times, notwithstanding the motion of the vessel. It usually beats half-seconds.

It is not possible to secure in the chronometer balance-wheel as perfect a temperature correction as in the pendulum, and for this and other reasons the best chronometers cannot quite com-

The observer, at the moment when a star crosses the wire, presses a key which he holds in his hand, and thus interpolates a mark of his own among the clock-beats on the sheet, — for instance, at X and Y in the figure. Since the beginning of each minute is indicated on the sheet in some way by the mechanism which produces the clock-beats, it is very easy to read the time

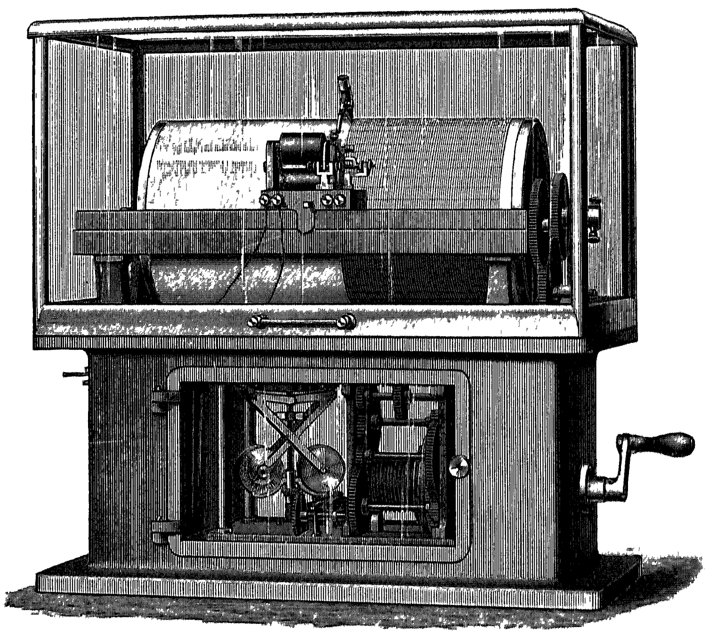


FIG. 26. A Chronograph

The clockwork of the chronograph is regulated by a centrifugal governor which acts continuously, not by beats like a clock escapement. (Built by Warner & Swasey)

of X and Y by applying a suitable scale, the beginning of the mark made by the key being the moment of observation.

74. The Transit Instrument. A large proportion of all astronomical observations for determining the position of a heavenly body are made when the body is crossing the meridian or is very near it. At that time the effects of refraction and parallax (to be discussed later) are a minimum; and as they act only vertically, they do not affect the *time* when a body crosses the meridian, or, consequently, its observed right ascension.

diameter, accurately round, without taper, and precisely in line with each other; in other words, they must be *portions of one and the same geometrical cylinder*. The fulfilling of this condition taxes the highest skill of the mechanician. The Y's must be worked with corresponding accuracy.

When exactly adjusted, the middle wire of such an instrument *affords a visible image of a part of the invisible meridian*, wherever the instrument may be turned on its axis; and *the sidereal time when a star crosses that wire is therefore the star's right ascension.*

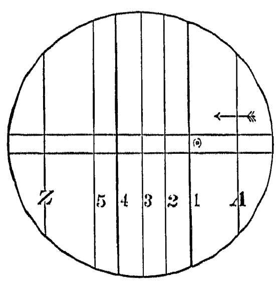


FIG. 28. Reticle of a Transit Instrument

The observation consists in noting the instant at which a star crosses each of the transit wires of the reticle When the chronograph is used, transits may be taken over wires which are quite close together (1 to 5) Eye-and-ear observations may be taken on wires A, 3, Z, the wider spacing of which permits the observer to make his record before the star reaches the next wire

76. Adjustments of the Transit. These are four in number:

(1) The reticle must be exactly in the *focal plane* of the object-glass, and the middle wire must be *accurately vertical.*

When the wires have been adjusted, the reticle slide should be tightly clamped and never disturbed again. The *eyepiece* can be moved to secure distinct vision for different eyes, reticle and star coming into focus together.

(2) The *line of sight* (that is, the line which joins the optical center of the object-glass to the middle wire) must be *exactly perpendicular to the axis of rotation.* It then coincides with the *line of collimation.* This may be tested by pointing on a distant mark and then reversing the instrument. The middle wire must still bisect the mark after the reversal; if it does not, the reticle must be adjusted by the screws provided for the purpose.

(3) The axis must be *level.* This adjustment is made mechanically by the help of the striding level, which may be set across the pivots. One of the Y's has a screw by which it can be slightly raised or lowered. When correct it is permanently clamped.

(4) The *azimuth* of the axis must be exactly 90°; that is, the axis must point exactly east and west. This adjustment is made by means of observations of a number of stars, by shifting one

are not very great, it is nevertheless possible to deduce from the observations themselves the true clock correction and the adjustment errors of the instrument.

The astronomer can never assume that *adjustments are perfect*; even if they were once perfect, they would not stay so, on account of changes of temperature and other causes. Nor are observations ever absolutely accurate. The problem is, from observations more or less *inaccurate* but *honest*, with instruments more or less *maladjusted* but *firm*, to find the result that would have been obtained by a perfect observation with a perfect and perfectly adjusted instrument. It can be done more nearly and more easily than one might suppose (see Campbell's *Practical Astronomy*), but the discussion of the subject belongs to practical astronomy.

78. Personal Equation. The Transit Micrometer. Even the best observers habitually note the passage of a star across the fixed wires of the reticle slightly too late or too early, by an amount which is different for each observer. This *personal equation* (though usually less than $0^s.1$ for chronographic observations) is an extremely troublesome error, because it varies with the observer's physical condition and also with the nature and brightness of the object. Faint stars are almost always observed too late, in comparison with bright ones; this gives rise to the so-called *magnitude equation.*

The effect of personal equation has been very greatly diminished by the introduction of the *transit micrometer*. In this instrument the reticle of fixed wires is replaced by a movable wire, carried by a micrometer screw which can be turned by hand, or by mechanism controlled by the observer, at any desired rate. The observer devotes his whole attention to keeping the moving star accurately bisected by this wire. An electric signal is automatically given whenever the screw reaches certain points in every revolution (that is, when the moving wire and the star bisected by it reach each of a definite series of positions in the field), thus furnishing an equivalent for the transits of the star over an equal number of fixed wires. The relative personal equations of observers, after a little practice with this apparatus, become almost vanishingly small and are almost independent of the brightness of the stars observed.

reaches the middle vertical wire marking the meridian. The utmost resources of mechanical art are expended in graduating the circle with precision. The divisions are now usually made either two minutes or five minutes of arc, and the further subdivision is effected by so-called reading microscopes, four of which, at least, are always used in the case of a large instrument (see Campbell's *Practical Astronomy*). By means of these microscopes the reading of the circle is made to a tenth of a second of arc.

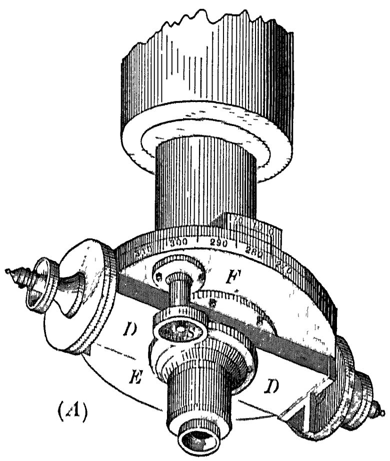
On a circle 2 feet in diameter, 1″ of arc is only about 1/17,000 of an inch; an error of that amount is now very seldom made by the best constructors in placing a graduation line. Even these minute *division errors* are carefully determined and allowed for by astronomers.

80. Zero Points. To reduce a circle reading to altitude or declination we must determine some *zero point* upon the circle, — the *nadir* point or the *horizontal* point if we wish to measure altitudes or zenith distances, the *polar* point or the *equator* point if we wish to measure polar distances or declinations. The polar point is determined by taking the circle reading for some star near the pole when it crosses the meridian above the pole, and then doing the same thing again twelve hours later when it crosses it below. The mean of the two readings (each corrected for refraction) will be the reading which the circle gives when the telescope is pointed exactly to the pole, — technically, the *polar point*. The *equator point* is, of course, 90° from the polar point.

The *nadir point* is the reading of the circle when the telescope is pointed vertically downward. It is determined by the reading of the circle when the instrument is set so that the horizontal wire of the reticle coincides with its own image formed by reflection from a basin of mercury placed on the pier below the instrument. To make this reflected image visible it is necessary to illuminate the reticle by light thrown toward the object-glass from behind the wires. The *zenith point* is just 180° from the nadir point thus determined.

81. Extra-Meridian Observations. Many objects are not visible when they cross the meridian: a comet, for instance, or a planet may be in such a part of the heavens that it transits only by day-

The box containing the wires can itself be rotated around the optical axis of the telescope so that the wires can be set in any desired position angle. By rotating the box, first, so that the wire passes through two stars, and, second, so that a star follows the wire exactly when the driving-clock is stopped, the direction of one star from the other can be measured. By turning the box until the wires are at right angles to the line joining the two stars the distance between them can be measured (§ 764).



The micrometer can also be used for measuring differences of right ascension and declination. It is widely employed in measuring the diameters of planets and satellites, and in determining the position of comets, the relative positions of close double stars, and the positions of faint objects near bright ones. For the last purpose it has

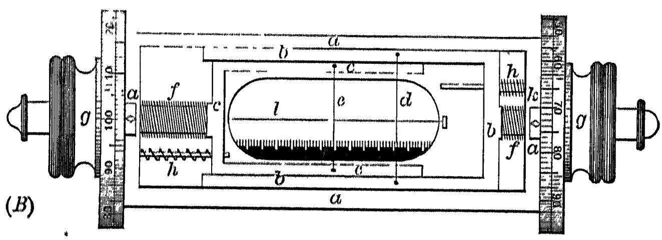


FIG. 32. The Filar Position Micrometer

Under the plate which carries the fixed wires lies a fork b moved by a carefully made screw with a graduated head g. This fork carries one or more wires parallel to the first set, so that the distance between the wires e and d can be varied and read off by means of the screw-head graduation

no rival. For the precise measurement of larger distances up to two or three degrees the heliometer was formerly used. That instrument is now only of historical interest.

83. Measurement of Photographs. For larger fields, or when many stars are to be observed in one region, photography now affords the most valuable method of measurement. With an

The images of bright stars or planets are often overexposed and cannot be measured with accuracy Again, even the smallest photographic star images are about ten times the diameter of the diffraction disks (§ 53) for the aperture employed. Close double stars and fine planetary detail must therefore be studied visually

84. The Sextant. All the instruments so far mentioned, except the chronometer, require some firmly fixed support and are therefore absolutely useless at sea. The *sextant* is the only one upon which the mariner can rely. By means of this he can measure the angular distance between two points (for instance, between the sun and the visible horizon), not by pointing first to one and afterwards to the other, but by sighting them both *simultaneously* and in *apparent coincidence*. A skillful observer can make the measurement accurately even when he has no stable footing.

The graduated arc of the instrument (Fig. 34) is usually, as its name implies, about a sixth of a complete circle, with a radius of about six inches. It is graduated in *half degrees* (which are, however, *numbered as whole degrees*), and it can measure any angle not much exceeding 120°. The *index-arm* is pivoted at the center of the arc and carries a *vernier*, which slides along the limb and can be fixed at any point by a clamp, with an attached *tangent screw* T. The reading of this vernier gives the angle measured by the instrument; the best instruments read to 10″.

Just over the center of the arc the *index-mirror* M, about 2 inches by $1\frac{1}{2}$ inches in size, is fastened to the index-arm, moving with it and keeping always perpendicular to the plane of the limb. At H the *horizon-glass*, about an inch wide and about twice the height of the index-glass, is secured to the frame of the instrument in such a position that when the vernier reads zero the index-mirror and horizon-glass will be parallel to each other. Only half the horizon-glass is silvered, the upper half being left transparent. E is a small telescope screwed to the frame and directed toward the horizon-glass.

If the vernier stands near but not exactly at zero, an observer looking into the telescope will see, together in the field of view, two separate images of the object toward which the telescope is directed; and if he slides the vernier, he will see that one of the images remains fixed while the other moves. The fixed image is

If the vernier does not read strictly zero when the mirrors are parallel, all the sextant readings will be too great (or too small) by a fixed amount. This *index correction* is, however, very easy to determine and allow for.

85. Observation with the Sextant. The principal use of the instrument is in measuring the altitude of the sun. At sea the observer usually proceeds as follows: Holding the sextant in his right hand, with its plane vertical, he points the telescope at the horizon vertically beneath the sun. Then he

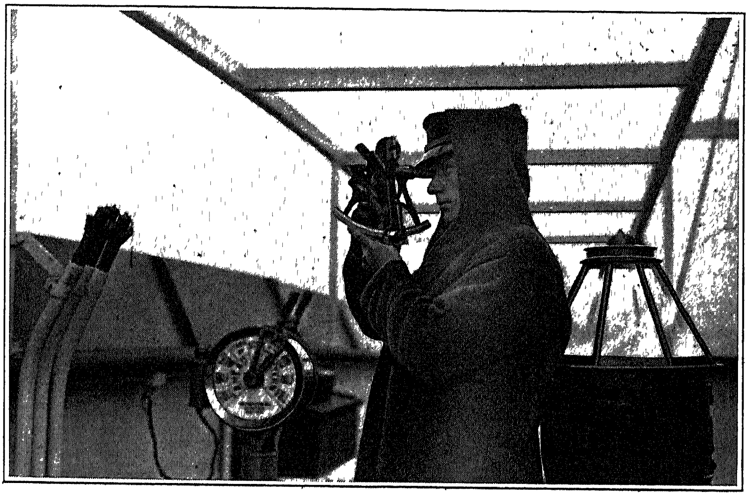


FIG. 35. "Shooting the Sun" with the Sextant

This photograph, taken on a destroyer, well shows the proper way to hold a sextant. The right hand grasps the handle The index finger of the left hand supports the arc, while the thumb and middle finger are used on the clamp and slow motion. (From an official photograph by the United States Navy)

slides the vernier along the arc with his left hand until he brings the reflected image of the sun down to the horizon. Tightening the clamp and using the tangent screw, he makes the lower edge, or limb, of the sun just graze the horizon as he swings the sun's image back and forth by a slight motion of the instrument, to make sure that he is measuring the vertical distance between sun and horizon. As soon as the contact is satisfactory he marks the time and afterwards reads the angle. The reading of the vernier, after due corrections (see Chapter III), gives the sun's true altitude at the moment.

The image of a star is usually brought to the horizon more expeditiously by pointing the telescope directly at the star, then moving the index-arm slowly forward as the telescope is lowered, thus keeping the star in the field

4. If a certain eyepiece gives a magnifying power of 60 when used with a telescope of 5 feet focal length, what power will it give on a telescope of 30 feet focal length?

5. What is the angular distance (theoretically) between the centers of two star disks which are just barely separated by a telescope of 24 inches aperture (§ 53)?

6. Why is it important that the two pivots of a transit instrument should be of exactly the same diameter?

7. If the middle wire in the reticle of a transit instrument is to the west of its proper position, what error in the observed time of transit will result? Will this error be the same for stars of all declinations? What will be the effect of reversing the instrument?

8. If the eastern Y is too far north, what will be the effect upon the transit of a star south of the zenith? north of the zenith? and how will these change with the declination?

9. What errors will result if the western Y is too high?

10. If the wires of a micrometer (Fig. 32) are set so that, used with a telescope of 10 feet focal length, a star moving along the right-ascension wire will occupy 15 seconds in passing from d to e, how long will it take when the micrometer is transferred to a telescope of 50 feet focus?

11. If the pitch of a micrometer screw is 1/75 of an inch, what is the angular value of one revolution of the screw when the micrometer is attached to a telescope of 30 feet focal length?

12. Does changing the eyepiece of a telescope for the purpose of altering the magnifying power affect the value of the revolution of the micrometer screw?

13. When the planet Mars is 20 seconds in apparent diameter, how large will its image be on a photograph taken with the Yerkes refractor of 19.36 meters focal length? *Ans.* 1.88 mm., or 1/14 inch.

REFERENCES

Louis Bell, *The Telescope* (McGraw-Hill Book Co., New York), a book "written for the many observers who use telescopes for study or pleasure and desire more information about their construction and properties."

The principal instruments employed in observations of position are described in detail, and their errors and adjustments discussed, in the works on practical and nautical astronomy Most of these treat also of spherical astronomy.

W. W. Campbell, *The Elements of Practical Astronomy* (The Macmillan Company, New York) — an excellent introduction.

William Chauvenet, *A Manual of Spherical and Practical Astronomy* (J. B. Lippincott, Philadelphia), an old but standard work, detailed and reliable, more advanced than the preceding, covering the field of practical astronomy as known in its day.

CHAPTER III

PROBLEMS OF PRACTICAL ASTRONOMY

FUNDAMENTAL AND DIFFERENTIAL METHODS • LATITUDE, LONGITUDE, TIME, RIGHT ASCENSION, DECLINATION, AND AZIMUTH • THE POSITION OF A HEAVENLY BODY • NAVIGATION • REDUCTION AND CORRECTION OF OBSERVATIONS • ERRORS OF OBSERVATION • COMPUTATIONS

87. There are certain problems of practical astronomy which are encountered at the very threshold of all investigations respecting the heavenly bodies, the earth included. The student must know how to determine his *position on the surface of the earth,* that is, his latitude and longitude; how to ascertain the *exact time at which an observation is made*; and how to observe the *precise position of a heavenly body* and fix its right ascension and declination.

The first astronomers had no body of collected data at their disposal; the observer had to rely, for the complete solution of his problem, entirely on his own observations, and devise his method accordingly. Such methods are called *fundamental,* because they are employed in laying the foundations of the science. They are still as important as ever, since each generation strives for an increase of accuracy on the work of its predecessors. Such methods are used to obtain the positions and motions of a sufficient number of fundamental objects, — the sun, moon, and planets, and a selected list of suitable stars. The possession of these data makes it possible to employ much less laborious methods in the great bulk of astronomical observation. In particular, the fundamental determination, with great precision, of the places of many hundreds of stars has made it possible, by measuring *differences* of right ascension and declination, to determine the places of many thousands of other stars. We shall follow the same order in our presentation of the methods of practical astronomy, — first the *fundamental* methods, then the *differential* methods.

strument is pointed to the star at X_2 and X_1 is the circle reading of the pole. The north polar distance, and hence the declination, of any other star is simply the difference of the circle readings on the star and on the pole.

In practice, however, it is found unsafe to rely on the stability of the instrument. The polar point of the circle may change from day to day and even from hour to hour. It is necessary to have some fixed, observable point of reference which may be set on at any time. The *nadir point* (§ 80) is the most convenient and satisfactory. Thus the direction of the force of gravity is actually utilized, after all.

The method fails for stations very near the equator, because there the pole is so near the horizon that the necessary observations cannot be made.

89. Time and Right Ascension. In practice the problem of determining time always consists in ascertaining the *correction of the timepiece*, that is, the *amount by which the clock or chronometer is fast or slow as compared with the time it should indicate.* The latter, or true time, must be found from observation.

The sidereal clock should indicate $0^h\,0^m\,0^s$ when the vernal equinox transits the meridian, but we cannot directly observe the equinox. When any star crosses the meridian, the sidereal-clock reading should equal the right ascension of the star, and the time elapsing between the transits of any two stars should equal their difference of right ascension. Until we know where the equinox is we cannot find the former, but we can observe the latter. Observations on successive days will show how much the clock is gaining or losing, but not how fast or slow it is. Correcting for the rate of the clock, we may therefore make a map or catalogue of all the observed stars with their correct declinations and correct differences of right ascensions from any arbitrarily chosen star, such as Sirius. We have now to put the equinox on this map. This is done by observing the sun in the same fashion as the stars, and thus determining its track (the ecliptic) in the heavens relative to the stars. If this is plotted on the map, its intersection with the equator will be the equinox. This could be done approximately by plotting on a globe, and much more accurately by computation.

DIFFERENTIAL METHODS

Latitude, Time, Longitude, and Azimuth

91. Latitude. Suppose we travel southward one degree of latitude; our zenith moves one degree southward among the stars, and our horizon tips down in the south and up in the north, rotating about the east-west line. The altitude of every star on the meridian to the south of the zenith increases by one degree, and the altitude of every star to the north decreases by the same amount. Since the separation of the two horizons is greatest at the meridian, an altitude measured here will distinguish much better between two such horizons (and determine the latitude more accurately) than the altitude of an object far from the meridian. When a star is on the meridian, its altitude depends only on its declination and on the latitude of the observer; when it is away from the meridian, its altitude depends also on the hour angle.

The general underlying principles of the many methods of determining the latitude from a measured altitude may be outlined as follows: The altitude is to be measured with a suitable instrument; the declination of the object — sun, moon, planet, or star — may be found in the *Nautical Almanac* or some other catalogue; and if we note the time at which the altitude is measured, we can find the hour angle of the object (§ 41). The latitude may then be calculated by a suitable formula.

92. Latitude by Meridian Altitude. In Fig. 36 the semi-circle $SQPN$ is the meridian, Z the zenith, P the pole, and Q the point where the equator crosses the meridian. QZ is the latitude (ϕ) of the observer — to be determined. If, when the star is on the meridian, we observe its zenith distance, Z_m (arc ZX in the figure), its declination δ (QX) being known, then evidently QZ equals QX plus XZ; that is, *the latitude of the place equals the declination of the star plus its zenith distance.* If the star is at X', south of the equator, the same equation still holds *algebraically*, because the declination (QX') is then a negative quantity. Therefore in all such cases (when the star and the pole are on opposite sides of the zenith) $\phi = \delta + Z_m = 90 + \delta - h_m$. When the star crosses the meridian between the zenith and the pole, as at X_2, the formula is

equal zenith distances of two stars which pass the meridian within a few minutes of each other, one north and the other south of the zenith, and not very far from it. Such pairs of stars can now always be found listed in our star-catalogues and almanacs. A sensitive level plays an important rôle in the measurement.

A special instrument, known as the *zenith telescope* (Fig. 38), is generally employed, though a simple transit instrument, provided with reversing apparatus, a delicate level attached to the telescope, and a declination micrometer, is now often used.

The telescope is set at the proper altitude for the star that first comes to the meridian, and the "latitude level," as it is called, which is attached to the telescope, is set horizontal. As the star passes through the field of view its distance north or south of the central horizontal wire is measured by the micrometer. The instrument is then reversed so that the telescope points toward the north (if it was south before), and, if necessary, the telescope so readjusted that the level is again horizontal. Great care must be taken, however, *not to disturb the angle between the level and the telescope itself*. Evidently the telescope is then elevated at exactly the same angle as before, but on the opposite side of the zenith. As the second star passes through the field we measure with the micrometer its distance north or south of the central wire. The comparison of the two measures gives the difference of the two zenith distances with great accuracy and *without the necessity of depending upon any graduated circle.*

In field operations, like those of geodesy, this is an enormous advantage, as regards both the portability of the instrument and the attainable precision of results.

By section 92 we have

for star *south* of zenith, $\phi = \delta_s + Z_s$;

for star *north* of zenith, $\phi = \delta_n - Z_n$.

Adding the two equations and dividing by 2, we have

$$\phi = \frac{\delta_s + \delta_n}{2} + \frac{Z_s - Z_n}{2}.$$

The *Nautical Almanac* gives the declinations of the two stars $(\delta_s + \delta_n)$; and the difference of the zenith distances $(Z_s - Z_n)$ is determined by the micrometer measures.

In some cases a person is so situated that it is necessary to determine the time roughly, without instruments; this can be done with an error less than about half a minute by establishing a noon-mark, which is nothing but a line drawn exactly north and south, with a plumb-line or some vertical edge, such as the edge of a door-frame or window-sash, at its southern extremity. The shadow will always fall upon the meridian line at *local apparent noon*.

95. Longitude. Having now the means of finding the true local time at any place, we can take up the problem of the longitude, the most important of all the economic problems of astronomy. The great observatories at Greenwich and Paris were established expressly for the purpose of furnishing the observations which could be utilized for its accurate determination at sea.

The longitude is simply *the difference between the local time and the Greenwich time at the same instant*. Since the observer can determine the former by the methods already given, the crux of the problem is to find, without leaving his place, the Greenwich local time corresponding to his own.

96. Telegraphic comparison may be made between his own clock and that of some station whose longitude from Greenwich is known. The difference between the two clocks will be the difference of longitude between the two stations after the proper corrections for *clock errors, personal equation, and time occupied by the transmission of the electric signals* have been applied.

The process usually employed is as follows: The observers, after ascertaining that they both have clear weather, proceed early in the evening to determine the local time at each station by an extensive series of star observations with the transit instrument. Then, at an hour agreed upon, the observer at the eastern station takes a telegraph key and makes a series of arbitrary signals, which are recorded, by means of relays, upon the chronographs at *both* stations. After fifteen or twenty such signals have been sent, the western observer takes his key and sends a series of thirty or forty signals; then the eastern observer sends another set like the first. The night's work is closed by another series of transit observations by each observer.

Upon each chronograph there are now the records of the clock times of sixty or more signals which would be simultaneous at the two stations if the transmission of the electric signals were instantaneous.

In practice the amount by which the western clock appears to be slow compared with the eastern clock will not be the same at the two stations. This discrepancy is evidently the sum of the times of transmission of the

A very precise determination of the difference of longitude between Paris and Washington was made by wireless in 1913–1914. Signals sent across the Atlantic in both directions indicated that the transmission time over the distance of 3830 statute miles was only $0^s.022$, giving a velocity of 175,000 miles per second, which agrees, within the (large) experimental error, with that of light.

98. Azimuth. An important problem, though one not so often encountered as that of latitude and longitude determinations, is that of determining the azimuth, or "true bearing," of a line upon the earth's surface.

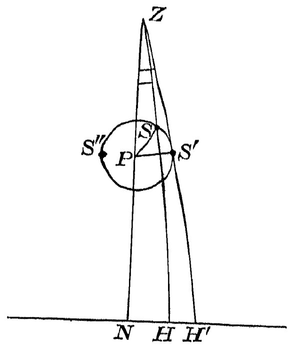


FIG. 39. The Azimuth of Polaris

When its hour angle is ZPS, the azimuth is the angle PZS, or the arc NH

With a theodolite having an accurately graduated horizontal circle the observer points alternately upon the polestar and upon a distant signal erected for the purpose, — usually an "artificial star" consisting of a small hole in a plate of metal, with a lantern behind it. At each pointing on the star he notes the time by a sidereal chronometer. The theodolite must be carefully adjusted for collimation (or reversed for the elimination of the collimation error), and special pains must be taken to have the axis of the telescope perfectly level. The next morning by daylight the observer measures the angle or angles between the night signal and the objects whose azimuths are required.

If the polestar were exactly at the pole, the mere difference between the two readings of the circle, obtained when the telescope is pointed on the star and on the signal, would give directly the azimuth of the signal. As this is not the case, the azimuth of the star must be computed for the moment of each observation, which demands only the solution of the "astronomical triangle" (Fig. 39, — P being, as usual, the pole, Z the zenith, NH' the northern horizon, and S the star) (compare § 42).

The polestar (or another fainter star close to the celestial pole) is used because a slight error in the assumed latitude of the place

a star-catalogue the observer, by limiting his attention to a very narrow *zone* of declination, is able to observe nearly all the brighter stars as they come to the meridian. In order to secure accuracy it is desirable that the observations should be repeated many times. Even in the best work, however, after correction for all known sources of error, there remain minute errors of obscure origin, varying with the right ascension, declination, and brightness of the star, which can be detected only by comparison with the results of other observers. The detection and determination of these "systematic errors" demand a high order of skill and judgment.

(2) *By the equatorial.* When a body (a comet, for instance) is too faint to be observed by the telescope of the meridian circle, which is seldom very powerful, or when it does not come to the meridian during the night, we must accomplish our observation with some instrument that can pursue the object to any part of the heavens. An equatorially mounted telescope is most suitable for the purpose.

100. With this instrument the position of a body is determined by measuring the *difference of right ascension and declination* between it and some neighboring star whose place has been accurately determined by the meridian circle and is given in a star-catalogue.

In measuring this difference of right ascension and declination a filar micrometer (§ 82) is usually employed, which is fitted with a number of fixed wires, set accurately north and south in the field of view, and one or more wires at right angles which can be moved by the micrometer screw. The difference of *right ascension* between the star and the object to be determined is measured by clamping the telescope firmly and then simply observing and recording upon the chronograph the transits of the two objects across the wires that run north and south; the difference of *declination* is measured by bisecting each object by one of the micrometer wires as it crosses the middle of the field of view. The difference of the two micrometer readings gives the difference of declination.

The observed differences must be corrected for refraction and for the motion of the body during the time of observation.

(we recall) to know three other elements — sides or angles. If we measure the *altitude* of an object of known *declination*, and know also its *hour angle* (and this requires a knowledge of the *longitude*), we may calculate the *latitude*; if we know the *latitude*, as well as the *altitude* and *declination*, of the body observed, we can calculate the *longitude*. We must know the latitude *or* the longitude in order to calculate the other. A single measured altitude will not give both the latitude *and* the longitude of a ship.

104. Dead Reckoning; Traverse Tables. By the older methods of navigation separate altitudes for latitude and longitude were measured several hours apart and each coordinate worked up with an assumed value of the other. These assumed values are obtained by *dead reckoning*. The change in latitude, for example, since the noon observation, may be calculated from the course and distance run, and used for the afternoon longitude observation. The actual calculation is avoided by the use of "traverse tables," which give the solution, in tabular form, of a right-angled plane triangle (Fig. 40). All three sides are measured in nautical miles, which are equal, very nearly, to minutes of arc of latitude. The change in latitude (DL), therefore, is given directly by the table. The *departure* (Dep.), however, which is the number of miles of *easting* or *westing* the ship makes, on the *course* steered, is not directly equal to the change of longitude, on account of the convergence of the meridians toward the poles; but the conversion is readily made by means of the traverse table used in a different way.

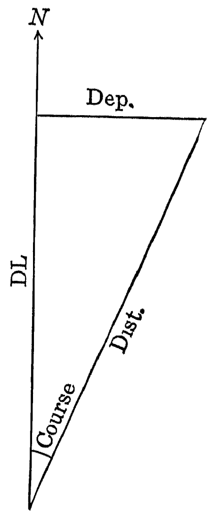


FIG. 40. Plane Sailing

105. Latitude by Meridian Altitude. The latitude is usually obtained by observing with the sextant the *sun's maximum altitude*, which occurs, of course, at *local apparent noon*.

If the navigator is somewhat uncertain of his position and does not know very closely the moment of local noon, he takes care

The method of noon altitude depends but indirectly upon the observer's knowledge of the time, which is used only in interpolating the declination from the *Almanac*. The declination of the sun changes most rapidly near the equinoxes, and then only at the rate of about 1′ per hour. An error of six minutes in the assumed time will produce, therefore, an error of only 10″ in the calculated latitude.

106. Latitude by Reduction to the Meridian and by a Single Altitude. If the observer knows his time with fair accuracy, he can obtain his latitude from altitudes measured *near* the meridian, applying to each the small difference, easily calculated from the hour angle, between it and the meridian altitude. By this method the observer is not restricted to a single observation at each meridian passage of the sun or of the selected star, but may utilize a considerable interval before and after this moment and gain the greater accuracy inherent in an average of several observations, or take advantage of a break in the clouds.

Finally, the altitude of an object some distance from the meridian may be measured, and the latitude calculated by a solution of the *ZPX* triangle. Obviously the method requires a more accurate knowledge of the time the farther the object is from the meridian, and is practically useless, when the sun is the body observed, beyond an hour angle of three hours. An observation of the slow-moving Polaris at any hour angle will yield a good value of the latitude.

107. Longitude by an Altitude of Sun or Star near the Prime Vertical. The observation consists in measuring the altitude and noting accurately the corresponding chronometer time. The declination is taken from the *Almanac*, and the latitude is assumed by dead reckoning. The hour angle t may then be computed. This hour angle, if the sun is observed, is the *local apparent time* at the moment of observation, and may be converted, by means of the equation of time, into *local civil time*. The difference between this time and that shown by the chronometer, at the moment of observation, is the longitude, provided the chronometer indicates true Greenwich civil time. As previously explained, the chronometer time will usually require correction for error and rate.

(1) Having observed an altitude at a known Greenwich time, assume a latitude north of the dead-reckoning position and compute the longitude from the observation. Repeat with the assumed latitude to the southward. Plot the two positions thus obtained, and the line joining them will be the Sumner line.

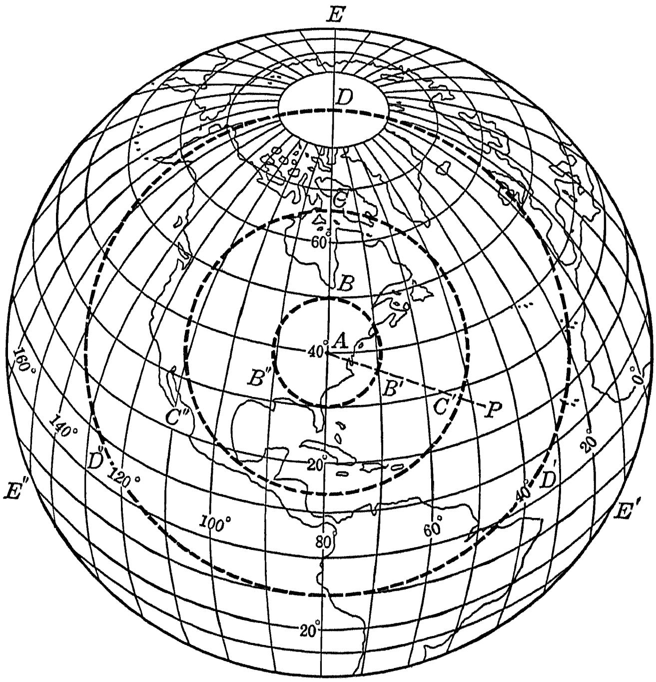


FIG. 41. Circles of Position

The observed body is vertically over A. Its declination is 40° North The hour angle of the body at Greenwich is $5^h 20^m$. The dotted circles are circles of position corresponding to measured zenith distances. P represents the assumed dead-reckoning position of the ship The observed zenith distance gives the improved position C', through which the Sumner line is to be drawn at right angles to the line $C'A$. (From the *American Practical Navigator*)

(2) A quicker method, due to Admiral St. Hilaire of the French navy, depends on the fact that *the direction of the Sumner line is at right angles to the bearing of the sun.* Starting with any convenient assumed position, we compute the sun's zenith distance and azimuth as seen from this point at the Greenwich time of observation. If this computed zenith distance agrees with the

which will give him his distance from the danger; and he will therefore observe the sun when it bears directly toward or away from the danger.

It should be noticed that in Sumner's method, as in all others, *the correctness of the position depends upon the accuracy of the Greenwich mean time given by the chronometer.* On this account the chronometer must be treated with great care.

THE REDUCTION OF OBSERVATIONS

Observations as actually made always require corrections before they can be used in deducing results. Those that depend on the errors or maladjustment of the instrument, which have already been referred to as belonging to the technical field of

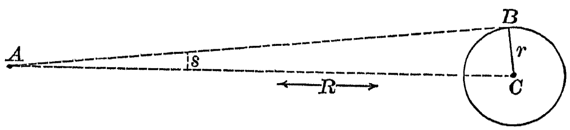


FIG. 42. Relation of Linear Diameter to Apparent Diameter and Distance

practical astronomy, will not be considered here, but only such as are due to other causes, external to the instrument (and the observer).

Thus far we have carefully avoided reference to the distance of the heavenly bodies; the discussion has been rather closely confined to their geometrical positions on the celestial sphere; but the element of distance enters into many of the methods of practical astronomy that have been described, and cannot be neglected.

109. Relation between the Distance and Apparent Size of an Object. The apparent size of an object depends upon its linear size and its distance from the observer; the larger it really is, and the nearer it is, the larger it will look.

Imagine a sphere having a (linear) radius BC equal to r. As seen from the point A (Fig. 42), its *apparent* (that is, angular) semidiameter will be BAC or s, its distance being AC or R.

From trigonometry, since B is a right angle, $\sin s = r/R$, whence also $r = R \sin s$, and $R = r/\sin s$.

of its center the angular semidiameter must be added or subtracted. For all objects except the moon this may be taken directly from the ephemerides, but the moon's apparent diameter increases slightly with its altitude, being about 1/60, or about 30″, greater when in the zenith than at the horizon, because at the zenith it is about 4000 miles, or 1/60 its whole distance from the center of the earth, nearer than at the horizon.

This augmentation is tabulated in works on navigation, and must be taken into account in accurate work. It has, of course, nothing whatever to do with the optical illusion, already referred to (§ 11), which makes the moon seem larger when near the horizon.

111. Parallax. In general the word "parallax" means the difference between the direction of a heavenly body as seen by the observer and as seen from some standard point of reference.

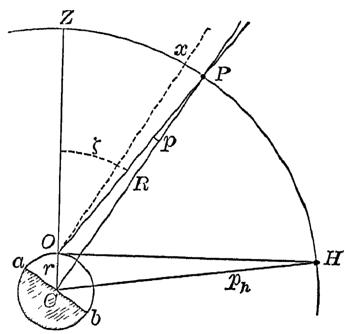


FIG. 43. The Geocentric Parallax

The *annual*, or *heliocentric*, parallax of a *star* is the difference of the star's direction as seen from the *earth* and from the *sun*. With this we have nothing to do for the present.

The *diurnal*, or *geocentric*, parallax of the sun, moon, or a planet is the difference of its direction as seen from the *center of the earth* and from the *observer's station* on the earth's surface; or, what comes to the same thing, it is the angle at the body made by two lines drawn from it, — one to the observer, the other to the center of the earth. In Fig. 43 the parallax of the body P is the angle OPC, which equals xOP and is the difference between ZOP and ZCP. Obviously this parallax is zero for a body directly overhead at Z, and a maximum for a body rising at H. Moreover (and this is to be especially noted), this parallax of a body at the horizon — the *horizontal parallax* — is simply *the angular semidiameter of the earth as seen from the body*. When we say that the moon's horizontal parallax is 57′, it is equivalent to saying that, seen from the moon, the earth has an apparent diameter of 114′.

The calculation of the parallax corrections to observations of the moon's right ascension and declination is also modified and greatly complicated (see Campbell's *Practical Astronomy*).

In the calculation of the parallax of all other bodies it is sufficient to regard the earth as spherical.

The parallax (4) is added to the measured altitude to reduce the latter to the earth's center. This correction is necessarily applied in order to prepare the altitude for use in formulæ involving almanac positions, which are always geocentric.

114. Refraction. The waves of light all travel with the same speed in empty space, but in transparent media, and particularly in air, their velocity diminishes by an amount which is proportional to the density of the air. If a wave passes through a region where the density is not uniform, the parts of it which move through the denser air may be supposed to lag behind those which traverse more rarefied air, and the *rays of light* (which are perpendicular to the wave-fronts) *will always be curved toward the region of greater density* (Fig. 44).

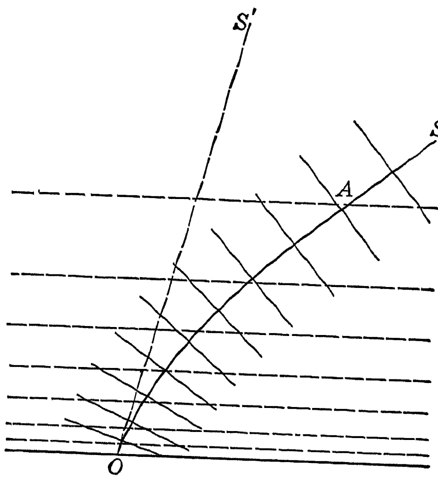


FIG. 44 Refraction of Light in the Earth's Atmosphere

Light from a star enters the atmosphere from the direction SA. Its velocity becomes less and less as it penetrates denser and denser layers. The direction of the wave-front changes more and more rapidly. The observer at O sees the star in the direction OS'. (The effect is much exaggerated here)

Since the density of the atmosphere increases downward, the rays from all heavenly bodies will be bent downward. We see them in the direction in which the rays enter our eyes, and *their apparent altitudes will exceed the true altitudes.*

Refraction, like parallax, will evidently vanish for rays which come vertically downward, and be a maximum for those which are nearly horizontal (Fig. 45). But the *law* of refraction is very

ing to the inclination of the sun's diurnal circle to the horizon, which varies with the time of the year. Sunset is delayed by the same amount, and thus at both ends the day is lengthened at the expense of the night.

Refraction at the horizon, for average conditions, and also semidiameter are taken into account in the computation of the times of sunrise and sunset. The problem is the same as that of finding the time by a single altitude of the sun. The zenith distance of the sun's center at the moment when its upper edge is rising (sunrise) equals, on the average, 90° 51′, that is, 90° plus 16′ (the mean semidiameter of the sun) plus 35′ (the mean refraction at the horizon). The *Nautical Almanac* contains tables of the times of rising and setting of the sun and moon. In the latter case the parallax is important, and direct calculation is rather complicated.

115. Mirage. When the surface of the land or the sea, and the air in contact with it, is considerably warmer than the air a few yards above, the density of the air may (locally) increase *upwards*. Rays of light passing through this region will be concave *upwards*. Under these conditions distant objects may appear greatly distorted, inverted, or suspended in the sky above the apparent horizon, producing the effects called *mirage* or looming, and the dip of the horizon may be quite abnormal. The image of the setting sun, especially when seen from a mountain, is often greatly distorted by similar anomalies of refraction.

116. Atmospheric Dispersion. The index of refraction of air, like that of almost all transparent bodies, is greater for green light than for red, and still greater for violet light. Hence a star at a low altitude, when observed with a high power, appears elongated vertically into a *spectrum*, with red at the bottom and green at the top (the blue being usually, and the violet always, lost by absorption in passing through so great a thickness of air). For the same reason the disk of a planet, or of the sun, seems to be bordered by a narrow red edge below and a green one above. This can be seen, even with the naked eye, at sunset, if the horizon is distant and sharp and the air perfectly clear. Just as the last glimpse of the sun disappears, its color changes from reddish yellow to green. Care must be taken, in observing this "green flash," that the eye is not fatigued by looking at the sun before it has almost completely disappeared.

the disk twinkles like a star, the different points do not keep step, so to speak, in their twinkling, and the general sum of light remains nearly uniform. When very near the horizon, however, the irregular refraction is sometimes sufficiently violent to make them dance and change color, -- especially in the case of Mercury, whose disk is very small.

118. Telescopic Effects; "Bad Seeing." These disturbances of refraction play havoc with telescopic definition. When a star is twinkling at all strongly, it appears in the telescope to dance madly about, and often, when the tremors are violent, to burst into an ill-defined mass of light many seconds of arc in diameter. The larger the telescope, the more pronounced are these effects. Such "bad seeing" is, next to actual cloudiness, the most serious of all hindrances to astronomical observation; and in most places really good nights, when the theoretical defining power of a great telescope can be approximately attained, are lamentably rare.

The most important of all factors in choosing the location of a great observatory is now recognized to be the *character of the seeing*. Mountains, high plateaus, and oceanic islands all have their advantages. Much depends on the nature of the work to be undertaken; the seeing at a given place may be good by day and poor by night, or vice versa.

The Lick Observatory on Mt. Hamilton, 40 miles from San Francisco and 4000 feet high, the Mt. Wilson Observatory, at an altitude of 6000 feet above Pasadena, and the Lowell Observatory at Flagstaff, at an altitude of 7200 feet, are noteworthy examples.

119. Twilight. Although this has nothing to do with the correction of observations, it is an atmospheric phenomenon and may most conveniently be treated here. It is caused by the *reflection* of sunlight from the upper portion of the earth's atmosphere. After the sun has set, its rays, passing over the observer's head, still continue to shine through the air above him, and twilight continues as long as any portion of the illuminated air remains in sight from where he stands. It is considered to end when stars of the sixth magnitude become visible near the zenith, which does not occur until the sun is about 18° below the horizon; but this varies considerably for different places, according to the purity of the air.

For *the distance of the sea horizon* (*OB*, Fig. 46) the approximate formula is

$$\text{Distance (miles)} = \sqrt{\tfrac{3}{2}\, h \text{ (feet)}}.$$

This, however, takes no account of refraction, and the actual distance is always greater.

121. Precession and Nutation. It is also in place to refer briefly to the corrections to a star's position which are made necessary by the motion of the coördinate systems. The equinoxes, the equator, and the ecliptic are all in constant motion (their motions are known as precessions and nutations, and are explained in § 165, p. 141); and so the right ascension, declination, etc. of every star is constantly changing Formulæ for allowing for this (which are quite complicated) are given in the *Nautical Almanac* and in star-catalogues.

DISCUSSION OF OBSERVATIONS: ERRORS, COMPUTATIONS

122. Errors of Observation. No actual observations can be made with absolute precision, the result of any measurement being more or less influenced by a multitude of circumstances. The study of these, and of the resulting *errors of observation*, forms an important part of all methods of precision.

Observational errors may be divided into two classes: (1) *systematic errors*, arising from causes which repeat themselves whenever the observation is repeated under similar conditions, and (2) *accidental errors*, arising from causes which do not so repeat themselves. The latter make the results of successive measurements of the same quantity differ slightly from one another, while the former affect all alike and can only be detected by repeating the observations under different conditions or by a different method. It is clear, therefore, that systematic errors are much more difficult to detect and troublesome to get rid of.

Systematic errors may arise from many causes: from peculiarities of the observer, such as personal equation (§ 78); from the unavoidable small imperfections of construction of an instrument, such as those of the transit (§ 77); or from external causes, such as refraction. In all work which aims at high precision it is necessary above all things to avoid systematic errors, either by

124. The Method of Least Squares. On account of the inevitable errors of observation it is impossible to obtain absolutely accurate values of any physical constant. To minimize the effects of such errors it is usual to make a large number of observations — much greater than would theoretically suffice if the measures were perfect — for the determination of the unknown quantities which we are seeking. In such a case the values of the unknowns are determined in such a way that the outstanding differences, or residuals, between the actual observations and the values calculated from these unknowns shall be as small as possible. More specifically, *the sum of the squares of the residuals must be a minimum.*

When the observations are successive, and presumably are equally accurate measures of the same quantity, this principle leads to the *arithmetical mean* of the individual observations as the best result which can be got from them. The probable error of this mean may be obtained by dividing that of one observation by the square root of the number of observations. It follows that the relative values, or weights, of observations of *different* probable error are *inversely proportional to the squares of their probable errors.*

The proof of this, and the discussion of the more complicated cases in which several unknown quantities have to be found from the same set of observations, may be found in treatises on the Method of Least Squares.

Probable errors are always written with the double sign $\pm$; for example, if a quantity was found to be $10''$, with a probable error of $2''$, it would be written $10'' \pm 2''$. It must always be remembered that the probable error of an observed quantity is a reliable measure of its precision *only* if the observations on which it is based are not affected by undetected systematic errors, and that there is an even chance, at best, that the observed quantity is wrong by more than the probable error.

125. Computations in Astronomy. It is already apparent, from the scope of the subject thus far covered, that the student of elementary astronomy should have some familiarity with the rudiments of trigonometry and with the use of logarithms. When calculations are to be made involving very large or very small distances, masses, or times, it will usually be found most convenient to express these in terms of centimeters, grams, and seconds,

1. Given the following meridian-circle observations on β Ursae Minoris at its upper and lower culminations respectively, namely.

Altitude 55° 48′ 6″.0, temperature 30° F., barometer 30.1 inches;
Altitude 24° 58′ 56″.4, temperature 25° F, barometer 30.1 inches.

The nadir reading (§ 88) was 270° 1′ 6″.8 in both cases. Required the latitude of the place and the declination of the star.

Ans. Lat. 40° 20′ 57″.8.
Dec. + 74° 34′ 40″.1.

2. Given the meridian altitude of the sun's lower limb, 62° 24′ 45″, the height of the observer's eye above the sea-level being 16 feet (§ 120). The sun's declination was + 20° 55′ 10″, and its semidiameter 15′ 47″. Its parallax at the observed altitude was 5″, and the mean refraction from Table 6 (Appendix) may be used. Required the latitude of the ship.

Ans. 48° 19′ 3″ N.

3. The meridian altitude of the sun, above the south horizon, is observed, on a ship at sea, to be 30° 15′ (after being duly corrected), the sun's declination at the time is 19° 25′ *south*. What is the ship's latitude?

4. At sea, the sun being on the meridian and south of the zenith, the altitude of its lower limb is observed to be 84° 21′. The sun's declination is + 18° 39′. Find the latitude.

5. Find the latitude from the meridian altitude of the moon's lower limb, 49° 37′, the moon being south of the zenith, its declination + 3° 13′, its semidiameter 15′.0, and its horizontal parallax 55′.2. The height of eye is 40 feet.

6. The *midnight* sun is observed on the meridian at a corrected altitude of 4° 11′. Its declination is + 22° 8′. What is the latitude?

7. Show by means of a Sumner line that a considerable error in the dead-reckoning latitude will make no difference in the longitude calculated from an observation of the sun when on the prime vertical.

8. On January 10, 1919, the moon's altitude was observed from an airplane with a bubble-sextant to be 38° 55′ at $8^h\ 37^m\ 33^s$ G M. T., and the sun's altitude to be 12° 59′ at $8^h\ 43^m\ 58^s$, — each observation being the mean of seven readings corrected for instrumental and refraction errors. The known position of the aircraft was 37° 4′ North, 76° 24′ West. The computed altitude of the sun at the time it was observed was 12° 55′, and its azimuth S 39 W, while for the moon the computed altitude (including the effect of parallax) was 39° 4′ and its azimuth S 81 E. Draw the Sumner lines and find the error of the position of the "ship" given by the observations.

Ans 9 nautical miles too far west and 2 miles too far north.

(These are actual observations made during experimental tests of navigating devices. See *Publications of the Astronomical Society of the Pacific*, June, 1919).

CHAPTER IV

THE EARTH AS AN ASTRONOMICAL BODY

ITS FORM, ROTATION, AND DIMENSIONS · THE EARTH'S MASS AND DENSITY · CONSTITUTION AND AGE OF THE EARTH

126. In a science which deals with the heavenly bodies it might at first seem to the student that the earth has no place; but certain facts relating to it are similar to those we have to investigate in the case of other planets and are ascertained by astronomical methods, and a knowledge of them is essential as a basis of all astronomical observations. Moreover, astronomical methods reveal important facts about the constitution and history of the earth which are not ascertainable otherwise. In fact, astronomy, like charity, begins at home, and it is impossible to go far in the study of the bodies which are strictly celestial until some accurate knowledge has been acquired of the dimensions and motions of the earth itself.

127. The astronomical facts relating to the earth are broadly these:

(1) The earth is a great ball approximately 7920 miles in diameter, and 24,880 miles in circumference.

(2) It rotates on its axis once in twenty-four sidereal hours.

(3) It is not exactly spherical, but is flattened at the poles, the polar diameter being nearly 27 miles, or about one three-hundredth part, less than the equatorial.

(4) Its mean density is between 5.5 and 5 6 as great as that of water, and its mass is represented in tons by 6 with twenty-one ciphers following (six thousand millions of millions of millions of tons), or 6×10^{21}.

(5) It is flying through space in its orbit around the sun with a velocity of about $18\frac{1}{2}$ miles a second, or nearly 100,000 feet a second, — about twenty times as fast as the swiftest modern projectiles.

the two terminal stations (*A* and *H*, Fig. 48) others are selected, such that the lines joining them form a complete chain of triangles, each station being visible from at least two others. The angles at each station are carefully measured, and the length of one of the sides, called the *base*, is also measured with all possible precision.

It can be done, and is done, with an error not exceeding half an inch in 10 miles (*BU* is the base in the figure) Having the length of the base and all the angles, it is then possible to calculate every other line in the chain of triangles and to deduce the exact *north-and-south distance* (*Ha*) between *H* and *A* An error of more than 3 feet in a hundred miles would be unpardonable.

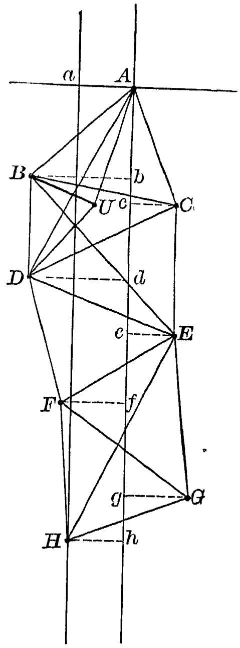


FIG. 48. Measuring the Earth's Diameter

Triangulation to find the linear distance between the two stations

130. The ancients understood the principle of the operation perfectly. Their best-known attempt at a measurement of the sort was made by Eratosthenes of Alexandria about 250 B.C., his two stations being Alexandria and Syene in Upper Egypt. At Syene he observed that at noon of the longest day in summer there was no shadow in the bottom of a well, the sun being then vertically overhead. On the other hand, the gnomon at Alexandria, on the same day, by the length of the shadow, gave him 1/50 of a circumference (7° 15′) as the distance of the sun from the zenith at that place This 1/50 of a circumference is therefore the difference of latitude between Alexandria and Syene, and the circumference of the earth must be fifty times the *linear distance* between those two stations.

The weak place in his work is the absence of details concerning the measurement of the linear distance between the two places. He states it as 5000 *stadia*. According to Dreyer the distance was probably measured in *paces* by specially trained men, and the stadium was 517 feet. This would make the earth's circumference 24,500 miles, — remarkably near the truth.

The first really valuable measure of an arc of the meridian was that made by Picard in northern France in 1671, — the measure which served Newton so well in his verification of the idea of gravitation. Since then many arcs of meridian have been accurately measured (§ 139).

rail some 12 feet across, with a little ridge of sand built upon it, was placed in such a way that a pin attached to the swinging ball would just scrape the sand and leave a mark at each vibration. To put the ball in motion it was drawn aside by a cotton cord and left for hours to come absolutely to rest; then the cord was *burned* and the pendulum started without jar to swing in a true plane.

But this plane at once began apparently to *deviate slowly toward the right*, in the direction of the hands of a watch, and the pin on the pendulum ball cut the sand ridge in a new place at each swing, shifting at a rate which would carry it completely around in about thirty-two hours if the pendulum did not first come to rest. In fact, the floor of the Panthéon was really and visibly turning under the plane of the vibrating pendulum.

A Foucault pendulum is of daily interest to visitors to the building of the National Academy of Sciences at Washington.

133. Explanation of the Foucault Experiment. The approximate theory of the experiment is very simple. A swinging pendulum, suspended so as to be *equally free to swing in any plane* (unlike the common clock pendulum in this), if set up at the pole of the earth, would appear to shift completely around in twenty-four hours.

It is easy to see that at the south pole the rotation will appear to be reversed. At the earth's equator there will be no such tendency to shift, while in any other latitude the effect will be intermediate and the time for the pendulum to complete the revolution of its plane will be longer than at the pole.

It can be proved that the hourly deviation of a Foucault pendulum equals 15° multiplied by the *sine* of the latitude. In the latitude of New York it is not quite 10° an hour.

The northern edge of the floor of a room in the northern hemisphere is nearer the axis of the earth than is its southern edge, and therefore is carried more slowly eastward by the earth's rotation. Hence the *floor* must *skew around* continually, like a postage stamp gummed upon a whirling globe, anywhere except at the globe's equator. The pendulum is constrained by the force of gravity to follow the changes in the direction of the vertical, but is otherwise free. Its plane of vibration, therefore, will appear to deviate in the opposite direction from the real skewing motion of the ground, and at the same rate. In the northern hemisphere it apparently moves in the same direction as the hands of a watch; in the southern hemisphere, in the opposite direction.

tion of the earth is not constant, but rather is very gradually decreasing, with a consequent lengthening of the sidereal day amounting to about 1/1000 of a second per century.

With the lengthening of the day the sun and moon *appear* to be moving faster *per day* than they used to. The cumulative effects of even this very minute change are clearly indicated by the comparison of modern with ancient eclipses. This has been suspected for a century, and has recently been shown conclusively by Fotheringham of Oxford.

The principal cause of this change is to be found in the friction of the tides (§ 355).

The sun, moon, and planets, during the last century, have shown a tendency to run ahead of their calculated positions in some years, and behind in others, in very much the way that would happen if the earth, considered as a clock, were sometimes as much as twenty seconds fast for a decade or two, and at other times slow, to the same maximum amount. E. W. Brown (1926) concludes that these changes of rotation are real and arise from alterations in the earth's diameter, due to internal causes, and amounting at most to a few feet.

137. Variation of Latitude; Motion of the Poles of the Earth. Any alteration in the arrangement of the matter of the earth must bring about a bodily shifting of the earth with respect to its axis of rotation. There results an apparent wandering of the terrestrial poles, which may be detected by a corresponding variation of the latitude of stations far from the pole.

The first satisfactory evidence of this fact was obtained at Berlin by Küstner in 1888, and at other German stations, and it has since been abundantly confirmed. Chandler found the same variations clearly exhibited in almost every extended body of reliable observations made since 1750. In 1900 a continuous series of observations for the study of the variations of latitude was started at six stations, distributed around the earth on the parallel of 39° 8′ north latitude, in Japan, Turkestan, Sardinia, Maryland, Ohio, and California. From the whole mass of evidence it is apparent that the movement of the terrestrial pole (Fig. 50) is composed mainly of two motions: one an *annual* revolution in a narrow ellipse about 30 feet long (as measured

III. THE EARTH'S FORM

As the earth rotates, every particle of it (except along its axis) is subjected to a centrifugal force directed perpendicularly away from the axis.

The vertical component (Fig. 51) of the centrifugal force neutralizes the force of gravity to a slight extent, which varies from point to point. The horizontal component, being directed toward the equator, acts to make the plumb-line hang away from the radius toward a point on the axis beyond the center. As things actually are, the earth is adjusted to bring the surface everywhere approximately perpendicular to the resultant direction of gravity, and takes, as a result of its rotation, the approximate form of an oblate spheroid (that is, an ellipsoid generated by revolving an ellipse about its minor axis).

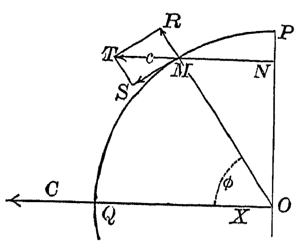


FIG. 51 Centrifugal Force caused by Earth's Rotation

138. Dimensions of the Earth and its Oblateness. The form of such a body is usually defined by its *oblateness f*, which is the fraction obtained by dividing the difference between the polar and equatorial semidiameters by the equatorial, that is,

$$f = \frac{a - b}{a}.$$

There are three ways of determining the form of the earth: First, by *geodetic measurement of distances upon its surface, in connection with the astronomical determination of the points of observation.* This gives not only the form but also the linear dimensions in miles or kilometers (§ 139).

Second, by observing the varying *force of gravity* at points in different latitudes; these observations are made by means of *a pendulum apparatus* of some kind, and determine *only the form*, not the *size*, of the earth (§ 151).

Third, by means of *purely astronomical phenomena*, known as *precession* and *nutation* (discussed in § 165 *et seq.*), and by *certain irregularities in the motion of the moon* (§ 342). These methods, like the pendulum method, give *only the form* of the earth.

pole. The length of a degree of latitude varies from 68.7 miles at the equator to 69.4 at the pole.

It will be understood, of course, that the length of a degree at the pole is obtained by extrapolation from the measures made in lower latitudes.

140. The deduction of the exact form of the earth from such measurements is an abstruse problem. Owing to local deviations in the direction of gravity, due to unevenness of surface and variation of density in the rocks near the station, the different arcs do not give strictly accordant results, and the best that can be done is to find the result which most nearly satisfies all the observations.

It is probably safe, judging by the probable errors of the observations, to put the practical limits of uncertainty of the value of the equatorial radius a, according to Hayford's results, as less than ± 150 meters.

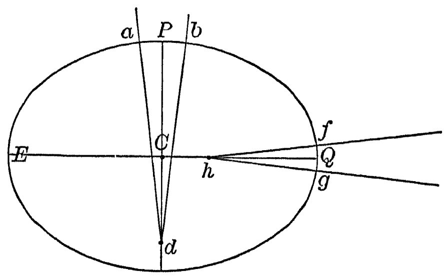


FIG. 52. Length of Degrees in Different Latitudes

The radius of curvature is longest at the pole. Therefore the degree of astronomical latitude is longest there

141. Arcs of longitude are also available for determining the earth's form and size. A degree of latitude is longer near the pole, and shorter near the equator, than on a sphere which has the same equator. Hence a point in a given astronomical latitude is farther from the earth's axis than it would be on the sphere, and a degree of longitude is longer. From this difference the oblateness can be computed.

In fact, *arcs in any direction between stations of which both the latitude and the longitude are known* can be utilized for the purpose, and thus all the extensive geodetic surveys that have been made by different countries contribute to our knowledge of the earth's dimensions.

142. Station Errors. If the latitudes of all the stations in a triangulation, as determined by astronomical observations, are compared with their differences of latitude, as deduced from the geodetic operations, we find discrepancies by no means insensible.

Geocentric latitude is employed in certain astronomical calculations, especially such as relate to the moon and to eclipses, in which it becomes necessary to "reduce observations to the center of the earth."

144. Surface and Volume of the Earth. The earth is so nearly spherical that we can compute its surface and volume (or bulk) with sufficient accuracy by the formulæ for a perfect sphere, provided we put the earth's *mean* semidiameter for radius in the formulæ.

This mean semidiameter of an oblate spheroid is not $\frac{a+b}{2}$ but $\frac{2a+b}{3}$, because if we draw through the earth's center three axes of symmetry at right angles to each other, only one will be the axis of rotation, and both the others will be equatorial diameters.

The *mean* radius r of the earth thus computed is 6371.23 kilometers, or 3958.89 miles; its surface ($4\pi r^2$) is 5.101×10^8 square kilometers, or 196,950,000 square miles, and its volume ($\frac{4}{3}\pi r^3$), 1.083×10^{27} cubic centimeters, or 259,000 million cubic miles, in round numbers.

IV THE EARTH'S MASS AND DENSITY

145. Mass, Volume, Density, Weight. It will be well, at this point, to remind ourselves, in a very elementary manner, of the proper meanings of such words as "mass," "density," etc. Very briefly, then: the *mass* of a body is the "quantity of matter" in it, and may be expressed in grams or pounds; its *volume* is the amount of space occupied by it, and may be expressed in cubic centimeters or cubic feet. *Density* is the mass contained in unit volume. The *weight* of a body is the force with which the earth attracts it. The childish conundrum "Which weighs more, a pound of feathers or a pound of lead?" illustrates the danger of confusing mass with density.

146. Gravitation. Science cannot yet explain "why" bodies tend to fall toward the earth, but Newton discovered that this phenomenon is only a special case of the much more general *law of gravitation: any two particles of matter attract each other with a*

tainable samples come from depths hardly more than 1/4000 of the distance to the center and are not typical of the whole earth. It is necessary, therefore, to use a wholly different method and to determine the mass of the earth by comparing the attraction which the earth exerts on a body, m, with the attraction exerted upon m by some other body of known mass at known distance.

148. The simplest method theoretically (and one capable of very considerable precision) is by the use of the common balance, first carried out by von Jolly at Munich in 1881.

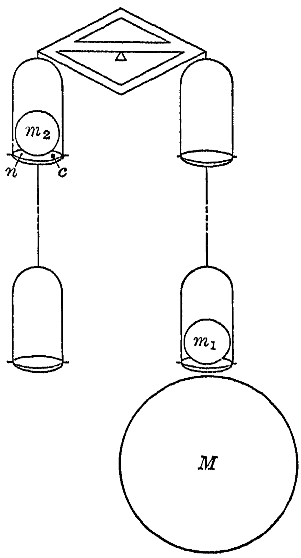


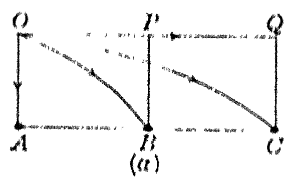
FIG 54. The Earth's Mass measured by the von Jolly Balance

An accurately constructed balance, capable of carrying considerable loads, is set up as illustrated diagrammatically in Fig. 54, with two sets of scale-pans, the lower hung from the upper by long wires. If two equal spherical masses, m_1 and m_2, are put in the two upper or the two lower pans, they will exactly balance, the earth's attraction on the two being equal. If m_2 is in the upper pan and m_1 in the lower, m_1 will be heavier than the other, since it is nearer the earth's center and more strongly attracted; but the balance can be restored by a small counterpoise c. If, after this is done, a large sphere of lead, of mass M, is brought underneath the lower pan, the equilibrium will again be disturbed because of the mutual gravitational attraction of the masses M and m_1, and an additional small mass n must be placed in the upper pan to restore the balance. Since the upper pan is so far from M that the attraction between the two is negligible, the attraction of M upon m_1 is equal to that of the earth upon n. But, by the law of gravitation, the first of these forces has the magnitude $G\frac{Mm_1}{d^2}$, where d is the distance between the centers of the spheres M and m_1; while the second is $G\frac{En}{R^2}$, where E is the mass of the

MEASUREMENT OF GRAVITY

149. The Law of Falling Bodies. In accordance with the principles of mechanics (§ 298) the acceleration a of a body moving under the influence of any force f (that is, the *rate at which it gets up speed* in the direction of the force) is given by the equation $a = f/m$, where m is the mass of the body. If the force is due to the gravitational attraction of a mass M at the distance R, $f = GmM/R^2$ (§ 146), whence $a = GM/R^2$. The rate at which the body falls depends, therefore, only on the mass of the *attracting* body, not on that of the one attracted.

If a body falls from rest, its downward velocity will be a at the end of one second, and at at the end of t seconds. Its average downward speed during this interval will be half as great, that is, $\frac{1}{2}at$. Multiplying this by the elapsed time, the distance fallen comes out $\frac{1}{2}at^2$. The acceleration at the earth's surface is denoted by g. The distance fallen in one second is approximately 16 feet, or 490 centimeters.



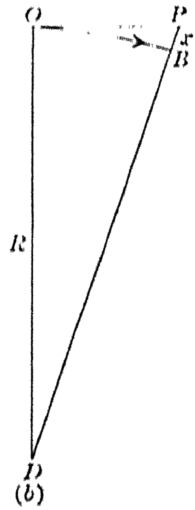


FIG. 55 Falling Bodies

From the point O (see a) a particle A is allowed to fall from rest, while two other particles B and C are simultaneously projected horizontally with velocities proportional to OP and OQ. At the end of t seconds A, B, and C will be on exactly the same level and directly underneath the points which they would have reached had there been no gravitational attraction, the distances OA, PB, and QC all being equal to $\frac{1}{2}at^2$. The distance that a particle falls under the action of a central force (see b) is likewise $\frac{1}{2}at^2$

If the particle does not fall from rest but is projected horizontally, its horizontal velocity will not be affected at all by gravity, nor will the amount by which it drops below the starting level be affected by its horizontal velocity (Fig. 55 a).

150. Centrifugal Force. We can now calculate the centrifugal force due to the earth's rotation. Suppose that a particle moves in a circle of radius R with uniform velocity v. If no force acted

about 1 part in 555 to be explained by the difference in the polar and equatorial radii. This difference amounts to more than thirteen miles. Since the earth is not spherical, the simple inverse square formula is not applicable, and a more complicated one must be used (§ 341).

152. Deflection of the Vertical; the Geoid. We are now prepared for a better understanding of station errors. Near the foot of a high mountain or a high plateau the plumb bob is attracted toward the excess of mass, and the direction of the zenith is deflected away from it.

One of the most striking examples of such deflection has been observed on the island of Porto Rico, which has very deep water to the north and south and is really the top of a great mountain range fully 20,000 feet high. The plumb lines of stations on the north and south coasts, 33 miles apart, are drawn together 56″. The surface of the ocean, therefore, slopes up toward the island, near which it is several feet higher than at a distance, when compared with a uniform spheroid.

The sea-level surface, to which measures of height are referred, is distorted all over the world. This surface, which is the one found by observation, is known as the geoid. It is found, however, that the geoid deviates but slightly (at most about 100 meters) from a smooth spheroid. The dimensions of the earth (§ 138) are the dimensions of this standard spheroid, and geographical latitudes (§ 143) are referred to it.

153. Isostasy. The most notable example of local attraction is to be found in northern India, where the enormous mass of the Himalayas and Tibet deflects the nadir strongly northward. A few hundred miles south of the mountains, however, the deflection has fallen off much more rapidly than the inverse square law predicts. This has been explained by the hypothesis that the earth's crust underneath the mountains is of lower density than elsewhere; the total mass per square kilometer to a moderate depth below sea-level is everywhere very nearly the same. When the observer is far enough away to make his distance from all points of such a column very nearly the same, the attraction of the column is almost exactly the same as it would be if the surface were at sea-level and the density uniform. Close to the foot of a

paths which they follow. Adams and Williamson have recently shown that the observed velocities cannot be harmonized on the assumption that the high density at the center is due solely to the diminution of volume by pressure.

It is necessary, therefore, to fall back on the second hypothesis, that the center is composed of denser materials. The occurrence of metallic iron in meteorites (§ 531) makes it probable that this dense substance is iron or an alloy of iron and nickel. This was first suggested by Wiechert in 1897. The latest evidence indicates that this metallic core is about 4000 miles in diameter, and of a density ten to twelve times that of water (which nickel-iron might attain under the great pressure). Adams and Williamson conclude that a layer of mixed iron and rock, about 1000 miles thick, surrounds the core (Fig. 56); Jeffreys (1926), that the transition to the rocky crust is abrupt. Most of this crust is composed of heavy basic rocks, and its mean density is about 4. The outer granitic layer, of density 2.7, appears to be only about 40 miles thick.

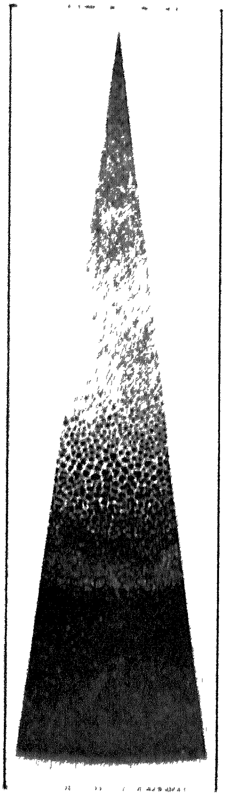


FIG. 56. What a Wedge Shaped Sample of the Earth Might Look Like

From an article by H. S. Washington, by courtesy of the *Scientific American*

(2) *The interior is solid and more rigid than steel.* This is proved by three independent lines of evidence:

(*a*) By the transmission of vibrations caused by earthquakes through the earth's interior to distant points. Investigations of this lead to the conclusion that the earth, taken as a whole, is considerably more rigid than steel. The inner core, however, does not transmit transverse vibrations, and is probably of low rigidity (Oldham, 1906). According to Jeffreys, it may actually be fluid. The rocky crust is very rigid.

the interior of the earth would be getting hotter. The rate of increase of temperature downward indicates, however, that radioactivity is largely confined to a thin superficial layer, and that it becomes insignificant at a depth of the order of 50 kilometers. Jeffreys concludes that the cooling of the earth, since solidification, amounts, at a depth of some 300 kilometers, to between 200° and 300°; at a depth of 700 kilometers the cooling is as yet insignificant; the physical state of the matter at great depths can scarcely have changed since the solidification of the earth.

155. The Age of the Earth. In recent years this problem, originally in the field of geology, has passed mainly into those of physics and astronomy, and more definite statements may be made concerning it than were formerly possible.

Much the best and most powerful line of attack is through the study of *radioactivity*. To state briefly a long and fascinating story, the heavy elements uranium and thorium disintegrate spontaneously but gradually, their atoms changing into atoms of quite different sorts (many of them, including radium, short-lived), but ultimately becoming atoms of lead. The lead produced from uranium has an atomic weight of 206; and that from thorium, 208. Both may thus be distinguished, by careful analysis, from ordinary lead, which is of atomic weight 207. When lead of this sort is found in a uranium mineral, it is reasonably certain that it has been formed by a radioactive change since the mineral crystallized from the melted rock. One per cent of the uranium is transformed in 66,000,000 years. In this way the ages of minerals in lower Pre-Cambrian rocks (the oldest geologically) from different parts of the world are found to be about 1,200,000,000 years. The earth's crust as a whole must be older than this.

On the other hand, a maximum age of the crust can be found from the relative proportion of uranium, thorium, and lead in its general composition (which is fairly well known from numerous rock analyses). It is thus found that all the existing lead would have been produced from the uranium and thorium in about 8,000,000,000 years. This does not, of course, date the creation of matter, but only the time within which the present crust was formed upon the planet. It appears likely, therefore, that the

CHAPTER V

THE ORBITAL MOTION OF THE EARTH

THE APPARENT MOTION OF THE SUN, AND THE ORBITAL MOTION OF THE EARTH • ABERRATION OF LIGHT • PRECESSION AND NUTATION • THE EQUATION OF TIME • THE SEASONS AND THE CALENDAR

156. The Sun's Apparent Annual Motion among the Stars. This must have been among the earliest recognized of astronomical phenomena, and it is obviously one of the most important.

As seen in the northern hemisphere, the sun, starting in the spring at the vernal equinox, mounts higher in the sky each day at noon for three months, until the summer solstice, and then descends toward the south, reaching in the autumn the same noonday elevation that it had in the spring. It keeps on its southward course to the winter solstice in December, and then returns to its original height at the end of a year, marking and causing the seasons by its course.

Nor is this all. The sun's motion is not merely north and south, but it also advances continually *eastward* among the stars. In the spring the stars rising on the eastern horizon at sunset are not those found there at that hour in summer or winter.

In March the most conspicuous of the eastern constellations at sunset are Leo and Boötes. A little later Virgo appears; in the summer, Ophiuchus and Libra; still later, Scorpio; while in midwinter Orion and Taurus are ascending as the sun goes down.

So far as the obvious appearances are concerned, it is quite indifferent whether we suppose the earth to revolve around the sun or vice versa. That it is the earth which moves, however, is demonstrated by three phenomena too delicate for observation without the telescope, but accessible to modern methods. The most conspicuous of them is *the aberration of light* (§ 162); the others are *the regular annual shift of the lines in the spectra of stars* (§ 732) and *the annual parallax of the stars* (§ 710).

we know the diameter in miles, yet the *changes in the apparent diameter* do inform us as to the relative distance at different times, — the distance being *inversely proportional* to the sun's apparent diameter (§ 109). If we divide 206,265 by the number of seconds in the sun's measured diameter at any date, we shall obtain (very approximately) the earth's distance from the sun, measured in *solar diameters* as units. If we lay off these distances on the arms of our spider, the curve joining the points thus obtained *will be a true map of the earth's orbit*, though without any scale of miles.

When the operation is performed, we find that the orbit is an ellipse of small eccentricity (about 1/60), with the sun not in the center but *at one of the two foci*.

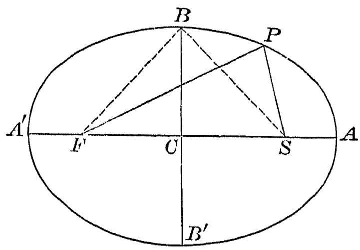


FIG 58. The Ellipse

$SP + FP = SB + FB = AA'$, the major axis AC is the *semi-major axis*, denoted by a. BC is the *semi-minor axis*, denoted by b. The *eccentricity* $e = SC/AC = \frac{\sqrt{a^2-b^2}}{a}$

159. Definitions relating to the Orbital Ellipse. *The ellipse is a curve such that the sum of the two distances from any point on its circumference to two points within, called the foci, is always constant and equal to the major axis of the ellipse* (Fig. 58).

Perihelion and *aphelion* are, respectively, the points where the earth is nearest to and remotest from the sun, the line joining them being the major axis of the orbit. The *line of apsides* is the major axis indefinitely produced in both directions. A line drawn from the sun to the earth or any other planet at any point in its orbit, as SP in the figure, is called the planet's *radius vector*, and the angle ASP, reckoned from the perihelion point, in the direction of the planet's motion, is called its *anomaly*. The mean of the perihelion and aphelion distances is called the *mean distance*. It is equal to half the major axis.

160. Discovery of the Eccentricity of the Earth's Orbit by Hipparchus. The variations in the sun's diameter are too slight to be detected without a telescope, so that the ancients failed to perceive them. Hipparchus, however, about 120 B.C., discovered that the earth is not the center of the

is the apparent displacement of a heavenly body, due to the combination of the orbital velocity of the earth with the velocity of light.

The fact that light is not transmitted instantaneously, but with a finite velocity, causes a displacement of an object viewed from any moving station, unless the motion is directly toward or from that object. The direction in which we point our telescope to observe a star is usually not the same as if we were at rest, and the angle between the two directions is the star's *aberration* at the moment (not to be confused with the aberration of lenses, § 54).

We may illustrate this by considering what would happen in the case of falling raindrops observed by a person in motion. Suppose the observer standing with a tube in his hand while the drops are falling vertically. If he wishes to have the drops descend through the tube without touching the side, he must obviously keep it vertical so long as he stands still; but if he advances in any direction, the drops will strike his face and he will have to draw back the bottom of the tube (Fig. 60) by an amount which equals the advance he makes during the time while a drop is falling through it; that is, he must incline the tube forward at an angle, a, which depends both upon the velocity of the raindrop and the velocity of his own motion, so that when the drop, which entered the tube at B, reaches A', the bottom of the tube will be there also. This angle is given by the equation

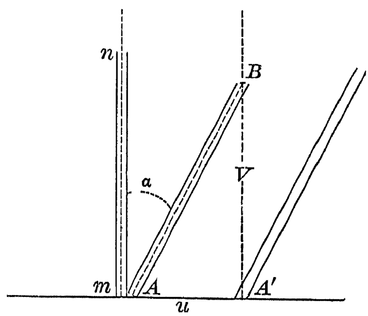


FIG 60. Aberration of a Raindrop

$$\tan a = u/V,$$

in which V is the velocity of the drop, and u the velocity of the observer at right angles to V.

This illustration is not a demonstration, because light does not consist of *particles* but of *waves* transmitted through space, but it can be shown that the apparent direction of motion of a wave is affected in precisely the same way. A discussion based on the principles of relativity, though more difficult, leads to the same result.

165. Precession of the Equinoxes. This is a slow westward motion of the equinoxes and was first discovered by Hipparchus about 125 B.C. He found that the "year of the seasons," from solstice to solstice (as determined by the gnomon), was shorter than that determined by the *heliacal rising and setting of the stars* (that is, the times when certain constellations rise and set with the sun), just as if the equinox "preceded," that is, "stepped forward" a little to meet the sun. The difference between the year of the seasons and the sidereal year is about twenty minutes. This difference of twenty minutes, since it is about one twenty-six-thousandth part of the year, can be accounted for by an annual westward motion of the equinox of about 50″ of arc ($1/26,000 \times 360°$). The annual precession in 1925, according to Newcomb, is 50″.2619.

Since the equinox is the point of intersection of the equator and the ecliptic, its motion must, of course, be interpreted as a motion of one or both of these circles and of their poles. As a matter of fact neither pole is stationary. That the motion of the pole of the equator (the celestial pole) contributes much the larger share of the precession is shown by the fact that the change in the latitudes of the stars in the last two thousand years has been very slight in comparison with the change in their longitudes, right ascensions, and declinations.

The motion of the *celestial pole* may be treated as partly periodic and partly progressive; that is, the actual pole may be considered as oscillating in a short period about a *mean pole* which moves steadily forward.[1]

The periodic motions of the celestial pole are known as *nutations*; the progressive motion of the mean pole, as (1) the *luni-solar precession.* The motion of the *ecliptic pole* produces (2) the *planetary precession,* and the sum of the two precessions is the *general precession.*

(1) *The luni-solar precession.* The mean pole moves around the pole of the ecliptic, regarded as fixed, in a circle at constant velocity. The equator continually shifts, therefore, so that its

[1] The distinction between progressive (often called secular) and periodic motions is not rigid, for the reason that the former, while apparently continuing indefinitely, may bring the object back to its initial position after the lapse of a very long period.

Aries is now in the *constellation* of Pisces, and so on. In the last two thousand years each sign has backed bodily, so to speak, into the constellation west of it.

The reader must again be warned against confusing the precessional motion of the *celestial* pole with the motion of the *terrestrial* pole which

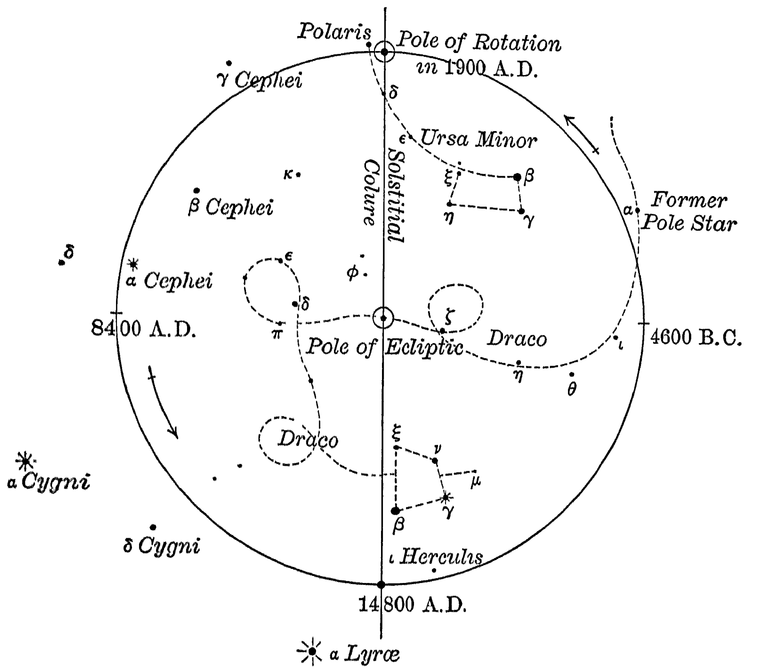


FIG. 62. Precessional Path of the Celestial Pole

Reckoning back about 4600 years, we see that α Draconis was then the polestar About 5600 years hence α Cephei will take the office, and about 12,000 years from now *Vega* (α *Lyræ*) will be the polestar — a splendid one, but rather far from the pole (Owing to the motion of the pole of the ecliptic the actual track of the celestial pole will not, however, be exactly circular, nor will it follow quite the same path in its next revolution)

causes the variation of latitude (§ 137). The former is a motion of the axis and the earth together; the latter is a motion of the axis within the earth. The latter involves a slight change in the bearing of one terrestrial object from another; the former does not, — a north-south line drawn on the earth's surface remains a north-south line notwithstanding the precession.

166. Physical Cause of Precession. The physical cause of this slow conical motion of the earth's axis was first explained by

planetary precession is the alteration of the plane of the earth's orbit by the action of the other planets.

167. Nutations. The forces which tend to pull the equator toward the ecliptic continually vary. When the sun and moon are crossing the celestial equator the action becomes zero, — twice a year for the sun, twice a month for the moon. Moreover, as we shall see (§ 188), the moon's orbit is continuously changing its position in such a way that the maximum declination attained by the moon during the month varies by as much as 10°. As a consequence the actual pole follows a sinuous curve, oscillating about the mean pole (steadily advancing by precession) in an irregular curve not very different from a circle. This involves an alternate motion toward and from the pole of the ecliptic (a nodding, which gives the motion its name of "nutation"), as well as a periodic variation in the rate of advance, sometimes known as the equation of the equinox. The largest nutation (that depending on the motion (§ 188) of the moon's nodes) has a maximum amount of 9″.21 and a period of a little less than nineteen years.

168. The Gyro-compass. This consists of a rapidly revolving gyro-wheel driven by an electric motor and so mounted that its axis is constrained to be horizontal but may move freely in a horizontal plane. If the axis points east and west, the earth's rotation tips one end down and the other end up (acting like the pull of the weight in Fig. 63) and produces a precession, which causes one end of the axis to seek the north and the other the south. If the axis overshoots the mark, the precessional force reverses in direction and brings it back. Since this instrument will work inside the armor of a battleship, which shields the ordinary compass against the earth's magnetic force, it is of great use in the navy. The directive force of the gyro-compass is many times that of the magnetic compass, and it points *true* north.

169. The Equation of Time. The equation of time at any moment is the difference between apparent and mean solar time, that is, the difference in hour angle of the sun and the fictitious mean sun (§ 36), and is therefore the difference of their right ascensions.

There are two principal causes of this difference:

(1) *The variable motion of the sun in the ecliptic, due to the eccentricity of the earth's orbit.* Near perihelion (about January 2)

so on. The maximum difference (in February, May, August, and November) amounts to 10 minutes.

170. Combination of the Effects of the Two Causes. We can represent the two components of the equation of time and the result of their combination by a graphical construction (Fig. 65).

The central horizontal line is a scale of *dates* one year long, the months being indicated at the top. The *dotted curve* shows that component of the equation of time which is due to the eccentricity of the earth's orbit. In the same way the *broken-line curve* denotes the effect of the obliquity of the ecliptic. The *heavy-line curve* represents the combined effect of the two causes,

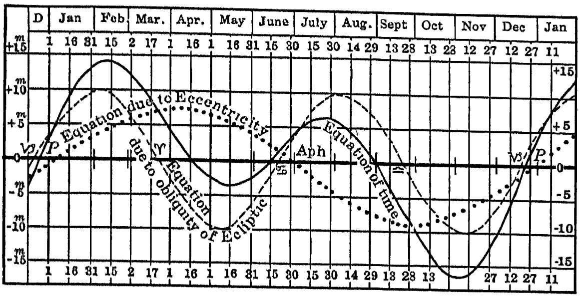


FIG. 65. The Equation of Time

Mean time minus apparent time. (The *American Ephemeris*, however, tabulates apparent minus mean.) According to the convention of this diagram the mean time is obtained from the apparent time by adding algebraically the equation of time to the latter

its ordinate at each point being made equal to the algebraic sum of the ordinates of the other two curves. The equation of time can be read from this curve within a minute, which is as closely as the apparent time can be found from a sundial.

The two causes discussed above are only the principal ones. Every perturbation suffered by the earth slightly modifies the result, but all other causes combined never affect the equation of time by as much as ten seconds.

The equation of time becomes zero four times yearly, as will be seen from the figure, — about April 15, June 14, September 1, and December 24; but the dates vary a little from year to year.

171. The Seasons. The earth in its orbital motion keeps its axis parallel to itself, except for the minute effect of precession. Since this axis is not perpendicular to the plane of its orbit, the

At all places within the *torrid zone*, which extends $23\frac{1}{2}°$ north and south of the equator and is bounded by the *tropics* of Cancer and Capricorn, the sun passes, at some time in the year, through the zenith. In the *temperate zones*, which are each 43° wide, the sun is never seen in the zenith, nor does it fail to appear above the horizon at noon. The *frigid zones* extend $23\frac{1}{2}°$ from the pole and are bounded by the arctic and antarctic circles. In these regions one or more days elapse in winter without the appearance of the sun, while in summer the sun makes one or more complete circuits above the horizon (§ 32 and Fig. 67).

172. Effects on Temperature. The changes in the *insolation* (exposure to sunshine) at any place involve changes of temperature and of other climatic conditions which produce the seasons. Taking as a standard the average amount of heat received from the sun in twenty-four hours on the day of the equinox, it is clear that the surface of the soil at any place in the northern hemisphere will receive, every twenty-four hours, more than the average of heat whenever the sun is north of the celestial equator, and for two reasons:

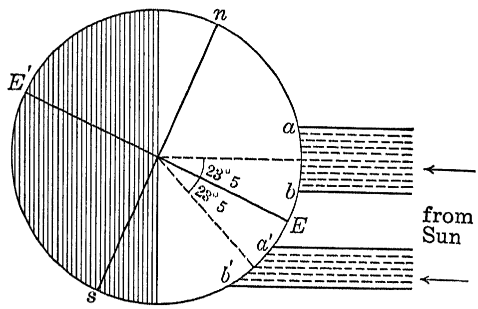


FIG. 68 Effect of Sun's Elevation on Amount of Heat imparted to the Soil

In June more heat from the sun reaches a given area (a to b) in the northern hemisphere than reaches an equal area (a' to b') in the same latitude south of the equator

(1) Sunshine lasts more than half the day.

(2) The *mean altitude* of the sun while above the horizon is greater than at the time of the equinox

Now the more obliquely the rays strike, the less heat they bring to each square inch of the surface (Fig. 68). A beam of sunshine of a certain cross-section is spread over a larger area when it strikes obliquely than when it strikes vertically, and its heating efficiency is in inverse ratio to the surface over which the heat is distributed.

revolution around the sun from a given direction in space to the same direction again.

The *tropical year* is the time included between two successive passages of the vernal equinox by the sun. On account of precession (§ 165) the equinox moves yearly $50''.3$ toward the west, so that the tropical year is shorter than the sidereal year, its length being $365^d\ 5^h\ 48^m\ 46^s.0$ ($365^d.24220$). Its length was determined by the ancients with considerable accuracy, as $365\frac{1}{4}$ days, by means of the gnomon (§ 90); they noted the dates at which the noonday shadow was longest (or shortest), that is, the dates of the solstices.

Since the *seasons* depend on the sun's place with respect to the equinox, the tropical year is the year of chronology and civil reckoning. Whenever a period of so many years is spoken of, we always understand tropical years unless the term is otherwise distinctly indicated.

A third kind of year is the *anomalistic year*, — the time between two successive passages of the perihelion. Since the line of apsides of the earth's orbit moves eastward about $11''$ a year (§ 328), this kind of year is nearly five minutes longer than the sidereal year, its length being $365^d\ 6^h\ 13^m\ 53^s\ 0$ ($365^d.25964$). It is very little used.

176. The Calendar. The natural units of time are the day, month, and year. The day is too short for convenience in dealing with considerable periods, — such as the life of a man, for instance, — and the same is true of the month, so that for chronological purposes the *tropical year* (the year of the seasons) is employed. At the same time so many religious ideas and observances have been connected with the changes of the moon that there used to be a constant struggle to reconcile the *month* (the period of the moon's orbital revolution) with the *year*. Since the two are incommensurable, no really satisfactory solution is possible, and the modern calendar of civilized nations entirely disregards the moon.

In ancient times the calendar was in the hands of the priesthood and was predominantly lunar, the seasons being either disregarded or kept roughly in place by the occasional intercalation or dropping of a month. The principal Mohammedan nations still use a purely lunar calendar for religious purposes, having a year of twelve lunar months, containing alternately 354

179. The change was immediately adopted by all Catholic countries, but the Greek Church and most Protestant nations refused to recognize the pope's authority. It was, however, finally adopted in England by an act of Parliament, passed in 1751, providing that the year 1752 should begin on January 1 (instead of March 25, as had long been the rule in England), and that the day following September 2, 1752, should be reckoned as the fourteenth instead of the third, thus dropping eleven days.

The change was bitterly opposed by many, and there were riots in various parts of the country in consequence, especially at Bristol, where several persons were killed. The cry of the people was, "Give us back our fortnight!" for they supposed they had been robbed of eleven days, although the act of Parliament was carefully framed to prevent any injustice in the collection of interest, the payment of rents, etc.

At present, since the years 1800 and 1900 were leap-years in the Julian calendar and not in the Gregorian, the difference between the two calendars is thirteen days. The Julian calendar was adhered to in Russia until 1918, and in Rumania until 1919, but both dates were customarily used for scientific purposes, for example, June 9/22, 1916.

When Alaska was annexed to the United States the official date had to be changed by only eleven days, one day being provided for in the alteration from the Asiatic reckoning to the American (§ 40).

180. The Julian Day. A system of chronological reckoning by days has many advantages in the simplification of calculations which involve long periods of time, and in the avoidance of ambiguity. According to the system proposed by J. Scaliger in 1582 a date is expressed as the number of days elapsed since the beginning of the arbitrary "Julian era," January 1, 4713 B.C. Thus, the date of the solar eclipse of January 24, 1925, is J.D. 2,424,175, and this is perfectly definite to every astronomer. The number of days between any two events, even centuries apart, is at once found by merely taking the difference between their Julian-day numbers. The *Nautical Almanac* gives the Julian-day number for January 1 of every year. By international agreement the Julian days still begin at noon.

18. Why do the afternoons begin to lengthen about December 8, a fortnight before the winter solstice?

19. There were five Sundays in February, 1880. When did this happen again? When will it happen again?

20. If the weather were "made on the spot where it is used and at the time when it is used," what would be the hottest place on the earth?

Ans. The south pole on December 22.

REFERENCES

SIR JOHN HERSCHEL, *Outlines of Astronomy* (D Appleton & Co , New York, 1876), gives a fuller explanation of the considerations on which the Julian-day system of reckoning is founded.

For those interested in the history of astronomy the following books are very pleasant and profitable reading:

MARY AGNES CLERKE, *Popular History of Astronomy during the Nineteenth Century.* A & C. Black, London.

ARTHUR BERRY, *A Short History of Astronomy* Charles Scribner's Sons, New York.

ROBERT GRANT, *History of Physical Astronomy to the Middle of the Nineteenth Century.* H. G Bohn, London

J. L. E. DREYER, *History of the Planetary Systems.* Cambridge University Press.

Since the moon moves eastward among the stars so much faster than the sun, it overtakes and passes the sun at regular intervals; and as its *phases* depend upon its apparent position with respect to the sun, this interval from new moon to new moon is especially noticeable and is what we ordinarily understand as the *month*, — technically, the *synodic month*.

The *elongation* of the moon is its angular distance from the sun at any time. When the moon has the same longitude as the sun, it is said to be in *conjunction* and is new; at full moon the

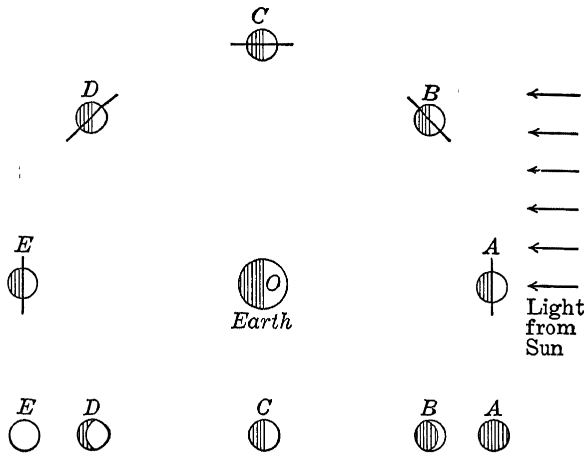


FIG. 69. Phases of the Moon

Sunlight from the right illumines one hemisphere of the revolving moon The corresponding phases apparent to the observer O, on the earth, are pictured below

difference in longitude is 180°, and it is said to be in *opposition*. In both cases the moon is in *syzygy*; that is, the sun, moon, and earth are ranged nearly along a straight line. When the elongation is 90°, it is said to be in *quadrature*.

183. The Phases of the Moon. Since the moon is an opaque body shining merely by reflected light, we can see only that hemisphere of its surface which happens to be illuminated, and of this hemisphere only that portion which happens to be turned toward the earth (Fig. 69) When the moon is between the earth and the sun (at new moon), the dark side is presented directly toward us, and the moon is entirely invisible. A week later, at

It is to be noticed that the straight line joining the ends of the terminator is always perpendicular to a line from the moon to the sun (Fig. 72 *B*), so that the *horns* are *always turned away from the sun.* The precise position in which they will stand at any time is perfectly predictable from the geometrical relations of the earth, sun, and moon. Artists sometimes carelessly represent a crescent moon at night with its horns pointed downward.

185. Earth-Shine on the Moon. Near the time of new moon the whole disk is easily visible, the portion on which sunlight does not fall being illuminated by a pale light (Fig. 71). This light is *earth-shine,* the earth as seen from the moon being then nearly full.

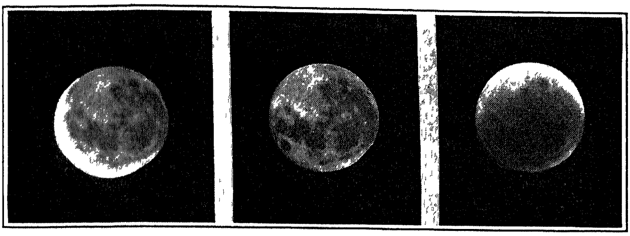


FIG. 71 The Earth-Lit, the Full, and the Totally Eclipsed Moon

That is, the moon illuminated by *reflected, direct,* and *refracted* sunlight. The picture on the left shows the "old moon in the new moon's arms" *sunlight* on the slender crescent, *earth-shine* on the rest of the moon In the middle is the picture of the full moon The picture on the right was taken during eclipse, although the moon is completely immersed in the earth's shadow, it is dimly illuminated by light *refracted* through the earth's atmosphere into the shadow The exposures necessary to obtain good photographs in the three cases were very different. (From photographs by E E. Barnard, Yerkes Observatory)

Seen from the moon, the earth would show all the phases that the moon does, the earth's phase being in every case exactly supplementary to that of the moon as seen by us at the time.

186. Sidereal and Synodic Months. The *sidereal month* is the time it takes the moon to make its revolution *from a given star to the same star again,* as seen from the center of the earth. It averages $27^d\ 7^h\ 43^m\ 11^s.47$ ($27^d.32166$), but it varies some seven hours on account of perturbations. The mean daily motion is $360° \div 27.32166$, or $13°\ 11'$. Mechanically considered, the *sidereal month* is the true month.

The *synodic month* is the time between two successive conjunctions or oppositions, that is, between successive new or full

The two points where the path cuts the ecliptic are called the *nodes*, the *ascending* node being the one where the moon passes from the south side to the north side of the ecliptic. The opposite node is called the *descending* node. (Ancient astronomers all lived in the northern hemisphere.)

On account of the so-called perturbations, due to the attraction of the sun, the moon at the end of the month never comes back exactly to the point of beginning.

One of the most important of these perturbations is the *regression of the nodes*. These slide westward on the ecliptic in the same manner as does the vernal equinox (§ 165), but much faster, completing their circuit in a little less than nineteen years instead of twenty-six thousand. The average time between two successive passages of the moon through the same node is called the *nodical* or *draconitic* month. It is 27.2122 days, — an important period in the theory of eclipses. The inclination also varies from $4° 59'$ to $5° 18'$, the mean being $5° 8'$.

When the *ascending node* of the moon's orbit coincides with the vernal equinox, the angle between the moon's path and the equator has its maximum value of $23° 27' + 5° 8'$, or $28° 35'$; nine and one-half years later, when the descending node has come to the same point, the angle is only $23° 27' - 5° 8'$, or $18° 19'$. In the first case the moon's meridian altitude will range, during the month, through $57° 10'$; in the second, through only $36° 38'$. The moon is much more effective in producing precession of the earth's axis in the first case than in the second. This accounts for the principal term in the nutation

189. Interval between the Moon's Successive Transits; Daily Retardation of its Rising and Setting. Owing to the eastward motion of the moon it comes to the meridian *later* each day. If we call the average interval between its successive transits a lunar day, we see at once that, while in the synodic month there are 29.5306 mean solar days, there must be just one less of these lunar days, since the moon, in the synodic month, moves around eastward from the sun to the sun again, thus losing one complete relative rotation.

It follows, therefore, that the length of the lunar day must be $24^h \times \frac{29.5306}{28.5306}$, or $24^h\ 50^m.47$. the average daily retardation of

lie still nearer the horizon than the ecliptic, and the phenomenon of the harvest moon will be especially noticeable.

191. Form of the Moon's Orbit. By observation of the moon's apparent diameter, combined with observations of its place in the sky, we can determine the *form of its orbit* around the earth in the same way as the form of the earth's orbit around the sun was worked out in section 158 (p. 136). The moon's apparent diameter ranges from 33′ 30″, when nearest, to 29′ 21″, when most remote.

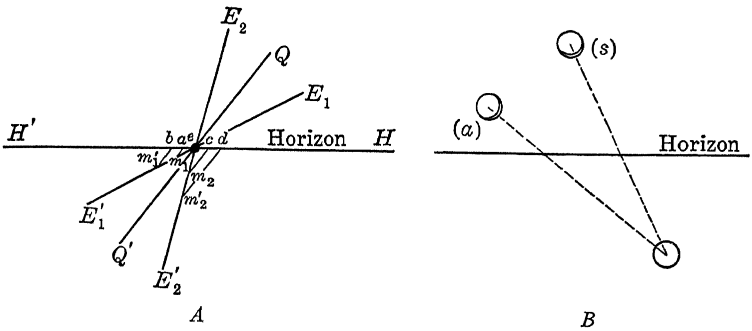


FIG 72. The Harvest Moon (A) and the Crescent Moon (B)

In the diagram at the left we are looking toward the east QQ' is the celestial equator, $E_1E'_1$ the moon's orbit (nearly coincident with the ecliptic) at the time of the full moon in September or early October. Suppose the moon rises at the east point e at 6 o'clock tonight. Tomorrow night at 6 o'clock it will be 12° 2 from e along its orbit at m_1 To reach the horizon it passes over the path m_1a (parallel to the equator), which is much shorter than m_1e The moon will rise (latitude 40°) at about 6 22. The next night it rises from m'_1 at 6 o'clock, to b at about 6 · 44, and so on. In March, when the moon's orbit occupies the position $E_2E'_2$, the retardation of rising is a maximum In the diagram at the right we are looking toward the west; the sun has set, but the crescent moon is still above the horizon The line joining the cusps of the crescent is at right angles to the line from the moon to the sun (§ 184), since the sun illuminates the hemisphere of the moon which faces it. In spring (s) the moon "holds water," and sets late; in autumn (a) it does not "hold water," and sets early. Of course these configurations are of no value in predicting whether the weather will be rainy or fair There is no scientific evidence that the moon influences either the weather or the growth of crops (§ 206)

The orbit turns out to be an ellipse like that of the earth around the sun, but of much greater eccentricity, averaging about 1/18. We say "averaging" because it varies from 1/15 to 1/23 on account of perturbations.

The point of the moon's orbit nearest the earth is called the *perigee* (περί + γῆ); that most remote, the *apogee* (ἀπό + γῆ). On account of perturbations the line of apsides is in continual

Knowledge of the law of the moon's orbital motion gives the ratio of the distance *OM* at this moment to the mean distance, which may be determined. The moon's parallax can also be deduced by means of occultations of stars observed at widely separated po: ı the earth and, most accurately of all, by gravitational . ɔry (§ 309).

193. Parallax, Distance, and Velocity of the Moon. The moon's *equatorial horizontal parallax* (at mean distance) is found to be 57′ 2″.7, according to Brown, but it varies considerably from day to day on account of the eccentricity of the orbit.

The corresponding mean distance of the moon from the earth is 238,857 miles, or 384,403 kilometers, or 60.267 times the earth's equatorial radius. The distance ranges between 252,710 and 221,463 miles.

Knowing the size and form of the moon's orbit, we can easily compute the mean velocity of its motion. It averages 2287 miles an hour, or about 3350 feet per second. The mean angular velocity in the celestial sphere is about 33′ an hour, just a little greater than the apparent diameter of the moon itself.

194. Form of the Moon's Orbit with Reference to the Sun. While the moon moves in a small elliptical orbit around the earth, it also moves around the sun in company with the earth. This common motion of the moon and earth does not, of course, affect their relative motion, but to an observer outside the system, looking down upon moon and earth, the moon's motion around the earth would be a very small component of the moon's whole motion as seen by him.

The distance of the moon from the earth is only about 1/390 of the distance of the sun. The speed of the earth in its orbit around the sun is also more than thirty times as great as that of the moon around the earth; for the moon, therefore, the resulting path *in space* is one which deviates very slightly from the orbit of the earth and is *always concave toward the sun.*

If we represent the orbit of the earth by a circle with a radius of 100 inches, the moon will deviate from it by only one fourth of an inch on each side, crossing it twenty-four or twenty-five times in one revolution around the sun, that is, in a year.

revolve together around this common center of gravity every month in orbits exactly alike in form but differing greatly in size, the earth's orbit being as much smaller than the moon's as its mass is greater.

The necessary result of this monthly motion of the earth's center is a "lunar equation," that is, a slight alternate eastward and westward displacement in the heavens of every object viewed from the earth as compared with the place the object would occupy if the earth had no such motion (Fig. 74). In the case of the stars or the remoter planets the displacement is not sensible, but this motion of the earth can be measured by observing through the month the apparent motion of the sun or, better, of one of the nearer planets, as Mars or Venus, or the recently discovered Eros, when nearest the earth.

From such observations it is found that the radius of the monthly orbit of the earth's center (that is, the distance from the earth's center to the common center of gravity of earth and moon) is 2880 miles. This is just about 1/82.5 of the distance from the earth to the moon, and by elementary principles of mechanics the conclusion follows that the *mass* of the moon is 1/81.5 that of the earth.

(2) The moon's mass may be found from the constants of precession and nutation; the mathematical analysis is difficult, but the results are accurate.

The most accurate determination of the moon's mass yet made is that derived by Hinks from observations of the small planet Eros, on the principle of method (1). He summarizes the results of various determinations of the ratio of the mass of the earth to that of the moon as follows:

Newcomb, from observations of the sun and planets, 81 48 $\pm$ 0.20
Newcomb, from the constants of precession and nutation, 81.62 $\pm$ 0.20
Gill, from observations of minor planets, 81.76 $\pm$ 0 12
Hinks, from observations of Eros, 81.53 $\pm$ 0.05
The weighted mean of these is **81.56 $\pm$ 0.04.**

Since the density of a body is equal to its mass divided by its volume, the density of the moon, compared with that of the earth, is found by dividing 1/81.56 by 0.0203. According to Ross the exact result is 0.6043 $\pm$ 0.0003 times the earth's den-

198. Geometrical Librations. While in the long run the moon keeps the same face toward the earth, this is not so in the short run; there is no crank connection between the earth and the moon, and the moon in different parts of a single month does not keep *exactly* the same face toward the earth, but rotates with perfect independence of her orbital motion. With reference to the center of the earth the moon's face is continually oscillating slightly, and these oscillations constitute what are called *librations*, (discovered by Galileo). We distinguish three, namely, the libration in *latitude*, the libration in *longitude*, and the *diurnal* libration.

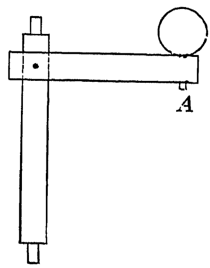


FIG. 75. The Moon's Rotation

(1) The *libration in latitude* is due to the fact that the moon's equator does not coincide with the plane of its orbit, but makes with it an angle of about $6\frac{1}{2}°$. This inclination of the moon's equator causes its north pole at one time in the month to be tipped $6\frac{1}{2}°$ toward the earth, while a fortnight later the south pole is similarly inclined to us, just as the north and south poles of the earth are alternately, for periods of six months, presented to the sun, causing the seasons.

(2) The *libration in longitude* depends on the fact that the moon's angular motion in its elliptical orbit is *variable*, while the motion of the rotation is *uniform*, like that of any other undisturbed body; the two motions, therefore, do not keep pace exactly during the month, and we see alternately a few degrees around the eastern edge and around the western edge of the lunar globe. This libration amounts to about $7\frac{3}{4}°$ each way.

(3) The *diurnal libration* Again, when the moon is rising we look over its upper edge, which is then its *western* edge, seeing a little more of that part of the moon than if we were observing it from the center of the earth; and vice versa when it is setting. This constitutes the so-called *diurnal libration* and amounts to about one degree. Strictly speaking, this diurnal libration is not a libration of the moon but of the observer. The telescopic effect is the same, however, as that of a true libration.

On the whole, taking all three librations into account, we see considerably more than half the moon, the portion that never disappears being about *41 per cent* of the moon's surface; that never visible, also *41 per cent*, while that which is alternately visible and invisible is *18 per cent.*

199. Physical Libration. Besides these geometrical librations, which arise from the lack of uniformity of the motion of the observer relatively to the

the moon cuts it. Further evidence of this sort comes from occultations of the stars. The star retains its full brightness in the field of the telescope until, all at once, without the least warning, it simply is not there, the disappearance generally being absolutely instantaneous. Its reappearance at the dark limb is of the same sort, and still more startling. Now if the moon had any perceptible atmosphere (or the star any sensible diameter) the disappearance would be gradual. The star-image would change color, become distorted, and fade away more or less gradually.

201. What Has Become of the Moon's Atmosphere? If the moon ever formed a part of the same mass as the earth, she probably once had an atmosphere. Its disappearance is explained on the basis of the kinetic theory of gases, according to which the molecules of a gas are continually flying in all directions with high velocities, colliding with one another and rebounding like perfectly elastic spheres. The mean-square velocity of the molecules (that is, the velocity whose square is equal to the mean of the squares of the individual velocities) varies inversely as the square root of the molecular weight of the gas, and directly as the square root of the absolute temperature. The value of this velocity at 0° centigrade is 1.84 km./sec. for hydrogen, 1.31 for helium, 0.62 for water vapor, 0.49 for nitrogen, 0 46 for oxygen, and 0.39 for carbon dioxide. At 100° C. these velocities are increased by 17 per cent.

Now, at any given distance from a body there is a so-called parabolic velocity, or velocity of escape, depending on the mass of the body (§ 314); and if a particle at this distance has a velocity, relative to the body, which is greater than the velocity of escape, it cannot be retained by the gravitational attraction but will fly off into space. At the surface of the earth the velocity of escape is 11.188 km./sec.; at the moon's surface, only 2.38 km./sec.; and at the sun's, 617 km./sec.

Even if the mean velocity of the molecules is considerably less than the parabolic velocity, the atmosphere will be gradually lost by the escape of fast-moving molecules from its extreme upper regions, where the free paths of the molecules are so long that they stand a chance of getting away without being stopped by collisions. It appears from the calculations of Jeans that if

The *brightness*, compared with that of sunlight, is difficult to measure accurately, and different investigators have found results for the ratio of sunlight to full moonlight ranging all the way from 375,000 to 630,000. The mean of the best determinations is 465,000, with a probable error of about 10 per cent. According to this, if the whole visible hemisphere of sky were packed with full moons, we should receive from it about one fifth of the light of the sun. The light of the full moon varies nearly 30 per cent with the changes in its distance. In comparison with artificial standards it is found that the light of the full moon is about one quarter as bright as that of a standard candle at a distance of one meter, or, in other words, that the intensity of full moonlight is 0.24 meter-candle (§ 568).

Photographically, full moonlight is only about 1/650,000 as bright as sunlight, which indicates that the moon's surface is yellowish.

After full moon the light falls off rapidly, the mean results of several observers being as follows:

Elongation	180°	160°	140°	120°	100°	80°	60°	40°	20°
Light	100	65	41	26	15	7 5	3 2	1.0	0 1

The waxing moon, shortly after the first quarter, is 20 per cent brighter than the waning moon at the corresponding phase before the third quarter, obviously because the region illuminated by the sun in the latter case contains more of the dark areas conspicuous to the eye.

The half-moon, though apparently of half the area of the full moon, is only one ninth as bright. Part of this difference arises from the fact that in the region near the terminator of the half moon the sun's rays strike the surface very obliquely, and therefore illuminate it feebly; but most of it must be due to the rough character of the lunar surface, which causes it to be more or less darkened, except at the full, by the shadows cast by its own irregularities. The shadows of the mountains which are visible with the telescope are probably of less importance than those of innumerable small irregularities, perhaps no bigger than bowlders or even pebbles. A homely illustration of the same principle is that a broken road of rough but white snow appears darker than

lunar day. Observation (§ 618) of the moon's heat confirms the high midday temperature, placing it at about 120° C.

206. Lunar Influences on the Earth. The moon's *attraction* coöperates with that of the sun in producing the *tides* (§ 345) and slight changes in the pressure of the atmosphere.

There are also certain distinctly ascertained disturbances of terrestrial magnetism connected with the approach and recession of the moon at perigee and apogee, but this ends the list of ascertained lunar influences.

The multitude of current beliefs as to the controlling influence of the moon's phases and changes upon the weather and the various conditions of life are unfounded, or at least unverified. It is quite certain that if the moon has any influence at all of the sort imagined, it is extremely slight, — so slight that it has not yet been demonstrated, though numerous investigations have been made expressly for the purpose of detecting it. It is not certain, for instance, whether it is warmer or not, on the average, or less cloudy or not, at the time of full moon.

207. The Moon's Telescopic Appearance and Surface. Even to the naked eye the moon is a beautiful object, diversified with markings which are associated with numerous popular myths. In a powerful telescope most of these markings vanish and are replaced by a multitude of smaller details which make the moon, on the whole, the finest of all telescopic objects, — especially so for instruments of moderate size (say from 6 to 10 inches in diameter), which generally give a more pleasing view of our satellite than instruments either much larger or much smaller.

An instrument of this size, with magnifying powers between 250 and 500, brings the moon optically within a distance ranging from 1000 to 500 miles; and since an object half a mile in diameter on the moon subtends an angle of about $0''.43$, it would be distinctly visible. A long line, or streak, even less than a quarter of a mile across can probably be seen. With larger telescopes the power can now and then be carried very much higher, and correspondingly smaller details made out, *when the seeing is at its best*, not otherwise.

For most purposes the best time to look at the moon is when it is between six and ten days old. At the time of full moon few

objects on the surface are well seen, as there are then no shadows to give relief.

It is evident that while with the telescope we should be able to see such objects as lakes, rivers, forests, and great cities, if they existed on the moon, it would be hopeless to expect to distinguish any of the minor indications of life, such as buildings (except perhaps the very largest) or roads.

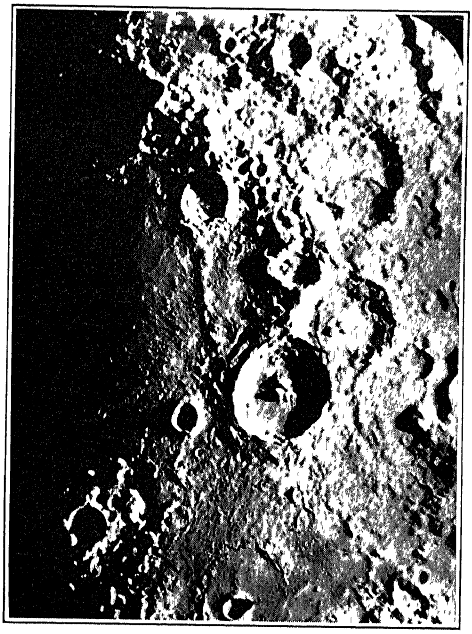


FIG. 78. Catharina, Cyrillus, and Theophilus

The great crater just below the center is Theophilus It is 64 miles in diameter, and the surrounding wall rises nearly 19,000 feet above the interior The central mountain is 5000 or 6000 feet high The floor lies several thousand feet below the level of the surrounding plain (From photograph by Yerkes Observatory)

208. The Moon's Surface Structure. The moon's surface is for the most part extremely broken. On earth the mountains are mostly in long ranges, like the Andes and Himalayas. On the moon the ranges are few in number; but, on the other hand, the surface is pitted all over with great *craters*, which closely resemble the volcanic craters on the earth's surface, though on an immensely larger scale. The largest terrestrial craters do not exceed 6 or 7 miles in diameter; many of those on the moon are 50 or 60 miles across, and some have a diameter of more than 100 miles, while smaller ones from 5 to 20 miles in diameter are counted by the hundred.

A typical lunar crater is nearly circular; the circumference is formed by a ring of mountains which rise anywhere from 1000 to 20,000 feet above the surrounding country. The floor within the ring may be either above or below the outside level; some craters

others, makes it natural to assume that they had a similar origin. This, however, is not absolutely certain, for there are considerable difficulties in the way, especially in the case of what are called the great Bulwark Plains. These lunar plains are so extensive that a person standing in the center could not see even the summit of the surrounding ring at any point; and yet there is no line of discrimination between them and the smaller craters; the series is continuous.

It is obvious, that if these lunar craters are the result of volcanic eruptions, they must be ancient formations, for it is quite certain that there is *no evidence of present volcanic activity.* The great plains, or *maria*, appear in some places to have invaded craters and broken down their walls; and it has been suggested that they were once actual seas of lava which melted their way into the adjacent mountains, the scattered craters within them being of subsequent formation.

210. Other Lunar Formations. The craters and mountains are not the only interesting formations on the moon's surface. There are a few long, straight lines of cliff of moderate height, which are evidently fault scarps, produced by motion along a crack in the crust, and not worn down by erosion like the thousands of similar features on the earth (Fig. 80, No. 9). There are many deep, narrow, crooked valleys called rills. Then there are numerous straight clefts, half a mile or so wide and of unknown depth, running in some cases several hundred miles, straight through mountain and valley, without any apparent regard for the accidents of the surface. They seem to be deep cracks in the crust of our satellite. Most curious of all are the light-colored streaks, or rays, which radiate from certain of the craters, extending in some cases a distance of many hundred miles. These are usually from 5 to 10 miles wide and neither elevated nor depressed to any considerable extent with reference to the general surface. Like the clefts, they pass across valley and mountain, and sometimes through craters, without any change in width or color. They have been doubtfully explained as a staining of the surface by vapors ascending from rifts too narrow to be visible.

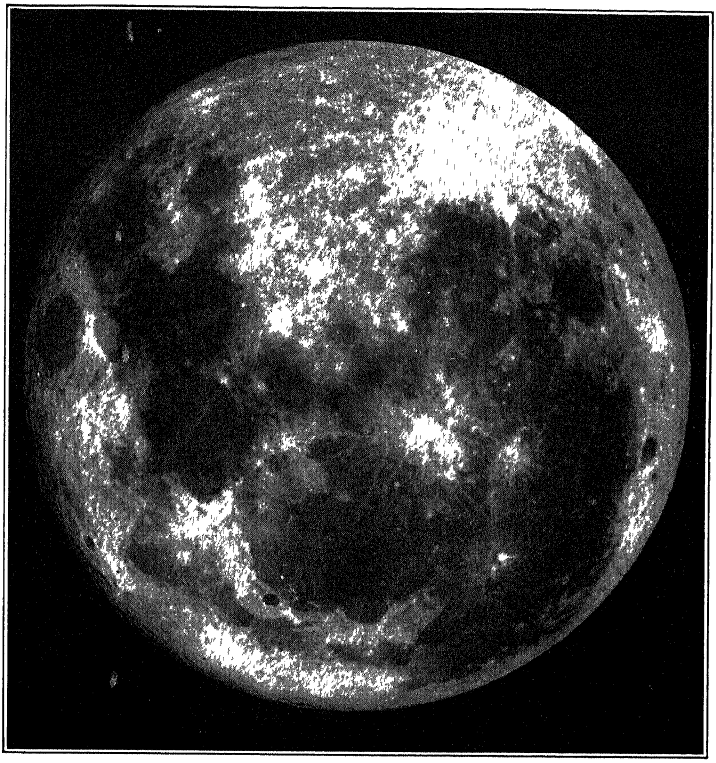


FIG 81. The Nearly Full Moon

With the surface in full sunlight there are no shadows to show the elevations, only a contrast in brightness Mountains and craters are brighter than the great plains Aristarchus is the brightest feature and Grimaldi the darkest The bright rays diverging from Tycho extend far round the globe, those about Copernicus are more intricate (Photographed at the Yerkes Observatory by Slocum)

days of full moon, but at that time they and the crater from which they diverge constitute by far the most striking feature of the whole lunar surface.

211. Lunar Photography. The moon was first successfully photographed by Bond in 1850, by means of the old daguerreotype process. Good results were obtained later by De la Rue in England and by Draper and Rutherfurd in this country; and further great advances have been made since 1890.

the same conditions in other respects, such as the height of the moon above the horizon and the clearness and steadiness of the air. It is, of course, very difficult to secure such identity of conditions. The disputed question whether short-lived changes, dependent on the phase of illumination, actually occur is obviously still more difficult to settle. No larger changes, such as might be caused by volcanic eruptions or landslides, have been detected since the advent of photography. The earlier drawings are not accurate enough to be good evidence.

213. Lunar Nomenclature. The great plains on the moon's surface were called by Galileo seas (*maria*), for he supposed that these grayish surfaces, which are visible to the naked eye and conspicuous in a small telescope, though not with a large one, might be covered with water.

Most of the ten *mountain ranges* on the moon are named after terrestrial mountains, as Caucasus, Alps, Apennines, though two or three bear the names of astronomers, like Leibnitz, Doerfel, etc.

The conspicuous *craters* bear the names of eminent ancient and medieval astronomers and philosophers, as Plato, Archimedes, Tycho, Copernicus, Kepler, and Gassendi, while hundreds of smaller and less conspicuous formations bear the names of more modern astronomers.

This system of nomenclature seems to have originated with Riccioli, who in 1650 made the first map of the moon.

214. Fig. 80 is reduced from a skeleton map of the moon by Neison and, though not large enough to exhibit much detail, will enable a student with a small telescope to identify the principal objects by the help of the key. (Compare with Fig. 81.)

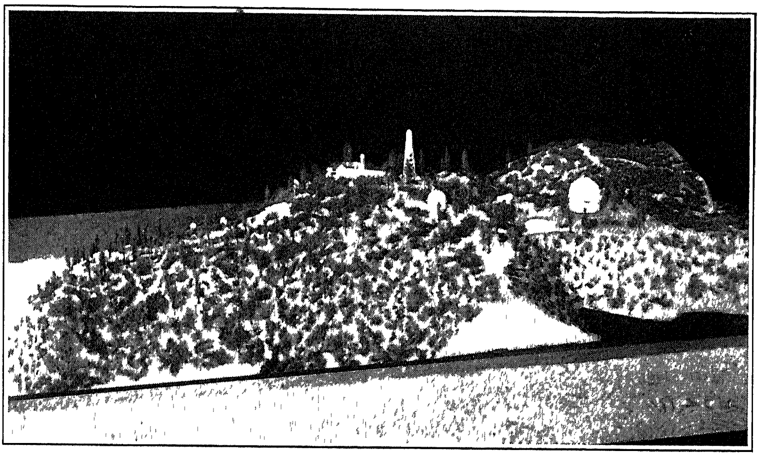
215. Lunar Maps. A number of maps of the moon have been constructed by different observers. A small map, such as Fig. 80 or the one in Schurig's *Himmels-Atlas*, is convenient for the identification of the formations. For precise purposes large-scale photographs completely supplant the older drawings. Our maps of the visible part of the moon are on the whole as complete and accurate as our maps of the earth, taking into account the polar regions and the interior of the continents of Asia and Africa

216. By photographing the moon with light of different colors, information can be obtained concerning the nature of the surface The photographs made by R. W. Wood with color screens and plates sensitive to various regions of the spectrum show decided differences in details of the lunar surface. The most conspicuous difference is in a spot close to the crater

13. When can the crescent moon be observed with its horns turned down?

14. Under what circumstances is the retardation of rising of the harvest moon the least possible?

Ans. The moon must be simultaneously at the vernal equinox, at the ascending node, and in apogee.



Mt. Wilson Observatory

From a model showing how the instruments are scattered on the forested top of the mountain The vegetation is carefully preserved to avoid heating of the soil and resulting air currents

218. Methods of Determining the Solar Parallax and the Sun's Distance. There are several very different ways of attacking this problem. We may classify under three general heads the methods that are capable of giving reliable results, simply referring here to the sections in which they are explained.

(1) Geometrical methods, which involve the direct measurement of the parallax of some planet or asteroid whose distance is known in terms of that of the sun (§ 287).

(2) Gravitational methods, which involve essentially the determination, from perturbations, of the ratio of the mass of the earth to that of the sun (§ 327).

(3) Methods depending upon the velocity of light, which give, directly, the earth's orbital velocity and the radius of the orbit (§ 164).

The best methods of each of the three classes yield very precise results.

VALUES OF THE SOLAR PARALLAX

METHOD	PARALLAX	PROBABLE ERROR
GEOMETRICAL METHOD		
Heliometer Observations of Asteroids, 1889–1890 (Gill) (§ 420) . .	8″ 802	± 0″.005
Visual Observations of Eros, 1900–1901 (Hinks) (§ 420) .	8″.806	± 0″.004
Photographic Observations of Eros, 1900–1901 (Hinks) (§ 420) . .	8″ 807	± 0″.0027
Photographic Observations of Mars, 1924 (Jones and Halm) (§ 287) . .	8″ 809	± 0″.005
GRAVITATIONAL METHOD		
Parallactic Inequality of the Moon's Motion, 1924 (Jones) (§ 337)	8″ 805	± 0″.005
Perturbations of Eros, 1921 (Noteboom) (§§ 420, 327)	8″ 799	± 0″ 001
VELOCITY OF LIGHT		
Radial Velocities of Stars, 1912 (Hough) (§ 732)	8″.802	± 0″ 004

The general mean of all these determinations is **8″.803 ± 0.001.** Other methods give closely accordant values. The results by the aberrational method (§ 164) are not included here, because this method necessarily involves the comparison of observations

the lunar motions. Fig 82 illustrates the size of the sun, and of such objects upon it as the sun-spots and prominences, compared with the size of the earth and the moon's orbit.

If we represent the sun by a globe 2 feet in diameter, the earth on the same scale would be 0 22 of an inch in diameter, the size of a very small pea, at a distance from the sun of 215 feet; *and the nearest star, still on the same scale, would be 11,000 miles away.*

Since the *surfaces* of globes are proportional to the *squares* of their radii, the surface of the sun exceeds that of the earth in the

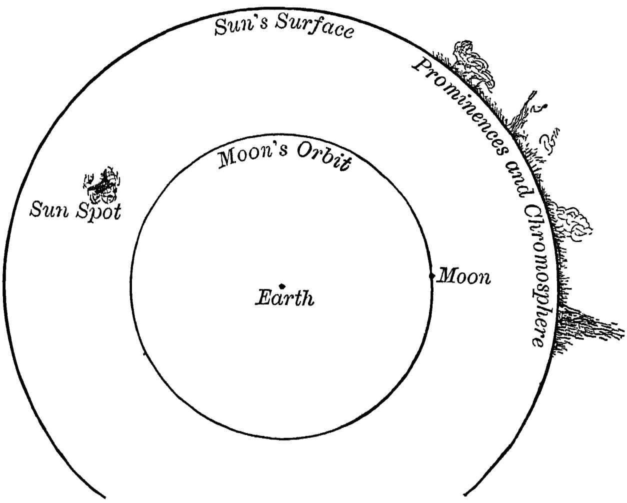


FIG. 82. Size of the Sun compared with that of the Moon's Orbit

ratio of $109.1^2 : 1$; that is, the area of the sun's surface is about 12,000 times that of the surface of the earth, or 6.075×10^{12} km.^{2}

The *volumes* of the spheres are proportional to the cubes of their radii. Hence, the sun's volume (or bulk) is 109.1^3, or 1,300,000 times that of the earth.

220. The sun's mass, like that of any other heavenly body, can be determined only by means of the effects of its gravitational attraction on some other body. It is obviously impossible to hold such a body fixed and measure the force with which the sun attracts it, but the acceleration in its motion (§ 149), produced by the sun's attraction, can be measured. The most convenient test object is the earth itself.

of consideration) *a watch* would go no faster there than here, since neither the *inertia* of the balance-wheel nor the *elasticity* of the spring would be affected by the increased gravity.

223. The Sun's Density. Its mean *density* as compared with that of the earth may be found by simply dividing its *mass* by its *volume*, or by the cube of its radius (both as compared with the earth); that is, the sun's density equals $331,950 \div (109.1)^3 = 0.256$, which is a little more than *one quarter* of the earth's density.

To get *its density compared with water* we must multiply this by the earth's mean density, 5.52, which gives 1.41; that is, *the sun's mean density is less than one and one-half times that of water.*

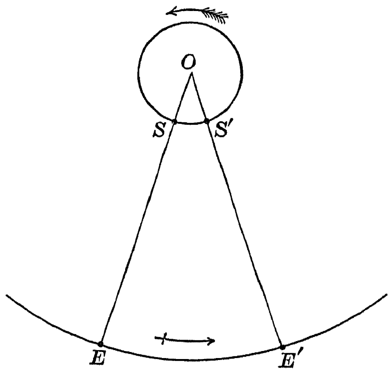


FIG. 83. The Synodic Period of Rotation is Longer than the Sidereal

An observer on the earth at E sees a spot on the central meridian of the sun at S, while the sun rotates, the earth also moves forward in its orbit, and when he next sees the spot in the same position on the disk of the sun he will be at E' The spot has gone around the whole circumference plus the arc SS'

This is a most remarkable and significant fact, considering that the sun has a tremendous force of gravity and that a considerable portion of its mass is composed of metals, as indicated by the spectroscope. The obvious and only possible explanation is that the temperature of the sun is so high that its materials are almost wholly in the gaseous state, — not solid or even liquid.

The pressure at the sun's center must be exceedingly great. For a sphere of uniform density of the sun's size and mass this pressure would exceed a billion atmospheres. If the density at the center is greater than the average, which is undoubtedly the case, the pressure must be still greater. To maintain the actual density at so great a pressure would demand a central temperature of many millions of degrees.

224. The Sun's Rotation and the Inclination of the Axis. The rotation of the sun is readily apparent to one who observes, from day to day, the positions of sun-spots on the solar disk. They

According to Mr. and Mrs. Maunder, who have investigated the motions of nearly 1900 spots shown on the Greenwich photographs of the years 1879 to 1904, the mean sidereal rotation period for spots on the equator is 24.65 days; in latitude 20°, 25.19 days; in latitude 30°, 25.85 days; and for the few in latitude 35° (beyond which there are hardly any spots), 26.63 days.

Individual spots show great deviations from these mean rates of motion, as if they were drifting backward or forward over the sun's surface; and the differences between the rotation periods derived from different spots in the same latitude extend over a range which considerably exceeds the difference of the mean periods for the highest and lowest latitudes. When, however, a large number of spots are considered, there is no doubt of the steady lengthening of the mean period on both sides of the solar equator.

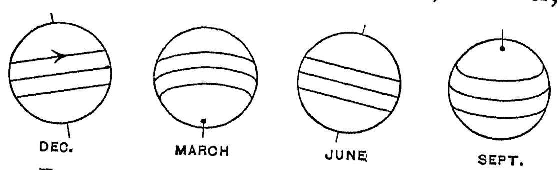


FIG. 85. Paths of Sun-Spots across the Sun's Disk

In September we see 7° beyond the north pole of the sun: the sun-spot paths are curved to the south. In March the south pole is tipped toward the earth and the sun-spot paths are curved to the north. In June and December the earth comes into the plane of the sun's equator and the spots seem to cross the disk in straight lines

226. If this remarkable equatorial acceleration were confined to the spots, it might be attributed to a drift of these over the sun's surface, like that of clouds over the earth; but there are several other ways of observing the sun's rotation, and the results of all agree closely. Long series of observations have been made on the *faculæ*, or bright regions on the sun's surface; on the *flocculi*, or regions of incandescent calcium vapor, revealed by the spectroheliograph (§ 584); and on the *gases of the lower portions of the solar atmosphere*, whose motion toward or from us may be measured spectroscopically (§ 579). The faculæ are confined to about the same latitudes as the sun-spots, and the flocculi extend but little farther from the solar equator; but the spectroscopic measures may be made at all solar latitudes, and show that the rotation grows steadily slower all the way to the pole. Thus Adams (1908) finds that the rotation period in latitude 40° is 27.48 days;

directly at it with a telescope. A very convenient method of exhibiting the sun to a number of persons at once is simply to attach to a telescope a small frame carrying a screen of white paper at a distance of a foot or more from the eyepiece (Fig. 86). When the focus is properly adjusted, a distinct image appears, which shows the sun's principal features very fairly, — indeed, with proper precautions, almost as well as could be done with elaborate apparatus. Still, it is generally more satisfactory to look at the sun directly with a suitable eyepiece.

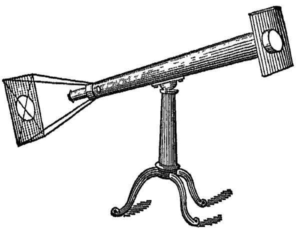


FIG. 86. Viewing the Sun
An image of the sun may be projected through the eyepiece upon a sheet of white paper. A screen should also be used at the object end, in order to shade the paper upon which the image is formed

With a small telescope (not more than $2\frac{1}{2}$ or 3 inches in diameter) a simple shade glass is often used between the eyepiece and the eye, but the dark glass soon becomes very hot and is apt to crack. With larger instruments it is necessary to use eyepieces especially designed for the purpose, and known as *solar eyepieces* or *helioscopes*, which reject most of the light coming from the object-glass and permit only a small fraction of it to enter the eye. It is not a good plan to cap the object-glass in order to reduce the light. To cut down the aperture is to sacrifice the definition of delicate details (§ 53).

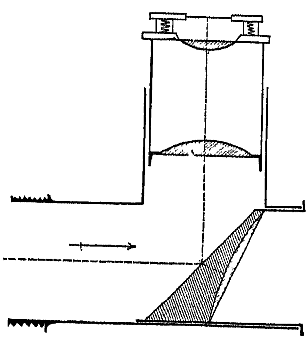


FIG. 87. The Herschel Solar Eyepiece

The simplest solar eyepiece, and a very good one, is known as Herschel's, in which the sun's rays are reflected at right angles by a plane of unsilvered glass. The reflector is made wedge-shaped, as shown in Fig. 87, in order that the reflection from the back surface may not interfere with the image. Most of the light passes through the open end of the eyepiece, but the reflected light is still too intense for the unprotected eye. Only a thin shade glass is required, however, which does not become very much heated. Polarizing eyepieces are also used.

(*b*) *the chromosphere* (composed mainly of the lighter gases, hydrogen and helium), which extends to a height of several thousand miles, and from which rise the *prominences* of various kinds. These are also masses of luminous gas, and rise sometimes to a height of hundreds of thousands of miles above the photosphere.

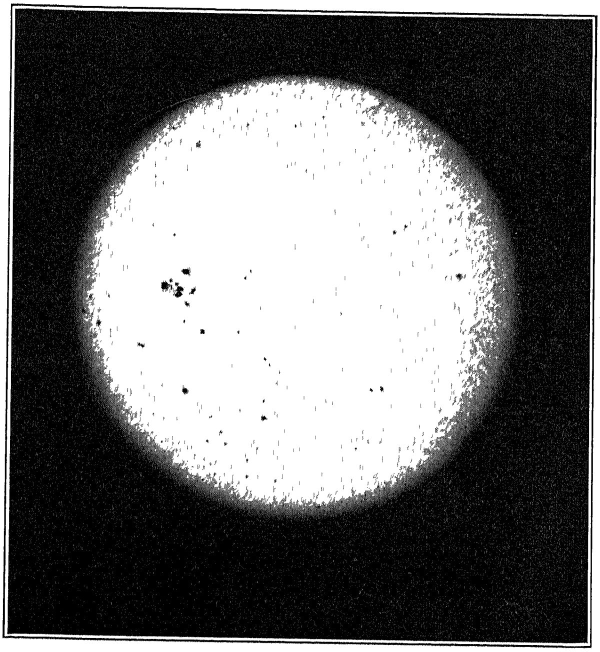


FIG 88 Sun-Spot Maximum

A large spot-group and many smaller spots Faculæ are conspicuous near the limb (From photograph by Yerkes Observatory)

(3) *The corona*, an outer envelope, of very great height and exceedingly small density, heretofore observable only during total eclipses of the sun.

230. The Photosphere. The sun's visible surface is called the *photosphere*, that is, the "light sphere." The general opacity (§ 659) of the gases in this region increases rapidly with depth and prevents us from seeing farther into the sun.

When studied with a telescope under favorable conditions and with a rather low power, it appears not smoothly bright, but mottled, looking much like rough drawing-paper. With a high

Certain bright streaks and patches called *faculæ* are also usually visible here and there upon the sun's surface, and, though not very obvious near the center of the disk, they become conspicuous near the limb, especially in the neighborhood of spots.

Near the sun's limb the photosphere appears much less brilliant than at the center. The variation is slight until near the limb,

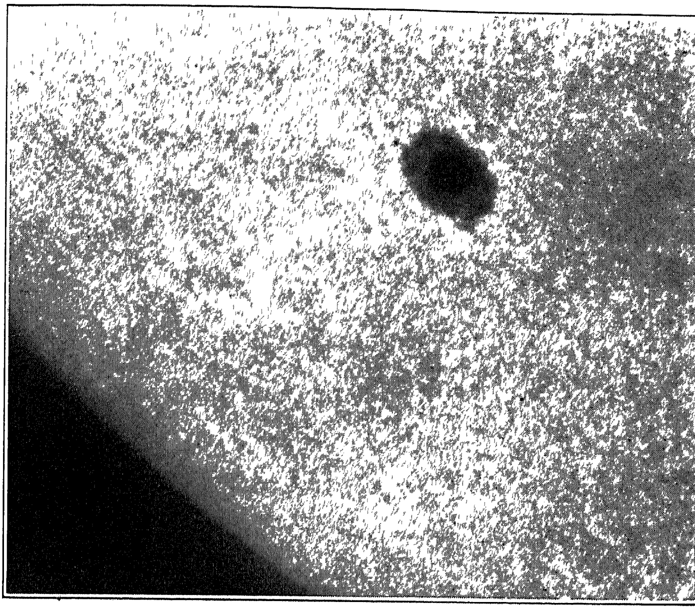


FIG. 90. Single Normal Spot, with Faculæ
From photograph by Mt Wilson Observatory

where the brightness falls off rapidly and, at the very edge, is not more than one third of the brightness at the center.

The light from the limb is not merely fainter but also redder than that from the center, so that on a projected image of the sun the limb appears brownish. For this reason the darkening at the limb is much more conspicuous on photographs than visually. When the sun is viewed with the naked eye through a shade glass, the contrast in brightness between the limb and the dark sky outside makes it appear relatively too bright and almost entirely conceals the darkening.

photosphere is usually much disturbed and brightened into faculæ, which ordinarily appear before the spot is formed and continue after it disappears.

Even the darkest portions of the sun-spot are dark *only by contrast*. This is directly observable when Venus or Mercury passes in front of the sun and transits across its disk. The planet (which, if there were no scattered light from the sky, would of course appear perfectly black) is conspicuously darker than the sun-spots. Further evidence that the spots give out light of their own is found in the fact that they are decidedly *redder* than the photosphere, and that their spectrum shows distinctive peculiarities.

Several lines of investigation indicate that the umbra of an average spot is rather less than one tenth as bright as the photosphere in yellow light, and relatively much fainter in the violet. Nevertheless, if we could see the spot on a dark background, instead of on the dazzling photosphere, it would appear brilliantly luminous, — brighter, indeed, for equal areas, than almost all terrestrial sources of light.

232. Dimensions of Sun-Spots. The diameter of the *umbra* of a sun-spot varies all the way from 500 miles, in the case of a very small one, to 40,000 or 50,000 miles, in the case of the largest. The *penumbra* surrounding a group of spots is sometimes 150,000 miles across, though that is exceptional. Not infrequently sun-spots are large enough to be visible with the naked eye and can actually be thus seen at sunset or through a fog or by the help of a shade glass.

The Chinese have many records of such objects, but interest in them dates from 1610, as an immediate consequence of Galileo's study with the telescope. Fabricius and Scheiner, however, share the honor of discovery with him, as independent observers.

233. Duration of the Spots. Most sun-spots are very short-lived phenomena. One fourth of all those shown on the Greenwich photographs lasted but a single day, and as many again, from two to four days. These, as might be expected, were small spots; the larger ones are far from permanent. Out of some 6000 groups observed in thirty-three years, only 468 were observed to have a continuous existence into a second rotation of the sun, 115 into

the sun's rotation) is usually more compact and regular, though the other is sometimes the larger. The leader apparently pushes forward upon the photosphere and so increases the length of the train of spotlets between the two principals. After a time these small spots generally disappear, leaving the pair, of which the leading one usually lasts the longer and looks more like a normal

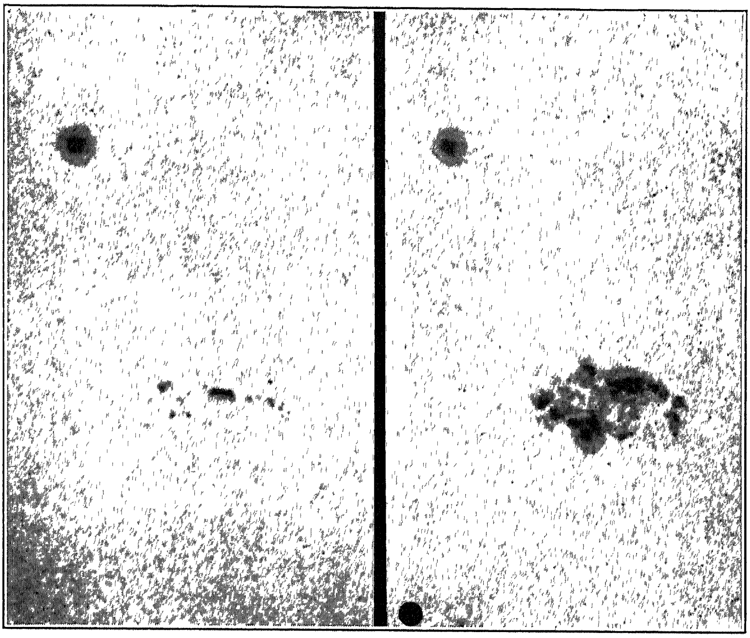


FIG. 93. Remarkable Twenty-four-Hour Development of Sun-Spot Group, August 18–19, 1916

The black disk at the bottom represents the relative size of the earth. (From photograph by Mt. Wilson Observatory)

spot. Recent studies at Mt. Wilson show that double groups are about twice as numerous as single spots. The leading member of a double group is usually the stronger (Fig. 84). Frequently a large spot divides into several, separated by brilliant bridges, and the segments fly apart with a speed of sometimes a thousand miles an hour. An active spot is an extremely interesting telescopic object; not infrequently a single day works a complete transformation (Fig. 93).

the French Academy as an anomaly During the last fifty years the southern hemisphere has been about one fifth more spotted than the northern, but the successive revivals of activity have come a little earlier in the latter. No reason for such differences has yet been discovered.

236. Sun-Spot Periodicity. The number of spots varies greatly in different years and shows an approximately regular periodicity of about eleven years. This fact was first discovered by Schwabe, of Dessau, in 1843, as the result of his systematic watching of sun-spots for nearly twenty years, and has since been abundantly confirmed.

Wolf, of Zürich, collected all the observations available up to 1880 and summarized them in a diagram which forms part of Fig. 94. The system of sun-spot numbers introduced by Wolf is based on the number of spot-groups, and on the number of spots which can be counted in these groups, as well as singly, and also takes into account the observer and the size of his telescope. The system has been continued by later observers. The numbers are found to be closely related to the spotted area expressed as a fraction of the visible hemisphere.

During the maximum, the surface of the sun is never free from spots; sometimes a hundred are visible at once. During the minimum, weeks, and even months, pass without a single spot.

The average interval between maxima is 11 13 years according to Newcomb; but this is subject to great fluctuations, the observed intervals ranging all the way from 7.3 to 17.1 years. The rise to maximum is usually, but not always, more rapid than the fall which follows, the mean durations of the two, according to Newcomb, being 4.62 and 6.51 years. W. J. S. Lockyer has pointed out that the rise is more rapid, both in actual duration and as compared with the fall, when the maxima are highest. The whole length of the interval between minima, however, seems to bear little relation to the intensity of the maximum.

There appears also to be a decided secular variation in the intensity of the outbreaks. High and low maxima, in groups of three or four, have alternated during the interval for which reliable estimates of the activity are available, and the fragmentary records of older times indicate a similar phenomenon. Sun-spots were apparently numerous about 1610 and very rare

appearance of small spots 25° or 30° north and south of the sun's equator. As the solar activity increases, new spots, in increasing numbers, supplant those which disappear, and the spotted zone widens out, but almost entirely on the equatorial side, so that the mean latitude of the spots steadily decreases. Shortly before the time of maximum activity the production of spots in the highest latitudes diminishes, and ceases, and not long afterward the inner edges of the two great zones of activity approach the equator and become stationary without quite meeting. As the activity decreases, the spotted regions shrink more and more on their outer edges, until the last dying traces of the disturbance are found within a few degrees on each side of the equator, thirteen or fourteen years after its first outbreak. Two or three years before this disappearance, however, the first spots of the new cycle show themselves near latitude $\pm 30°$, so that, at the spot minimum, there are usually four well-marked zones in which the few spots may appear, — two close to the equator, owing to the expiring disturbance, and two in high latitudes, nearly 200,000 miles away from the former, owing to the newly beginning outbreak.

Fig. 95, reproduced from a paper by Maunder, shows the distribution of the spots in heliographic latitude for each synodic rotation of the sun from 1874 to 1903, and exhibits the facts better than any verbal description could do. It is clear that if we confined our attention to a zone of definite latitude, we should always find approximately the same intervals between the times of greatest abundance of sun-spots, but that the times of maximum would differ widely from zone to zone, — maximum for the highest latitudes and minimum for the equator being almost simultaneous.

238. Irregularity of the Period of Sun-Spots. A great deal of labor has been expended in the attempt to find a mathematical formula which shall represent the changes of the observed numbers of spots in the past and may be used to predict those in the future. No formula which will stand the test of prediction has yet been obtained, and it appears very doubtful whether success is possible, for it is not unlikely that the sun-spot disturbances, like the eruptions of a geyser, are inherently only roughly periodic.

out as the last is dying away — points to a deep-seated cause, perhaps involving a great portion of the mass of the sun. Of the nature of this cause we can form but the vague conception that it is a gathering of deep-lying forces during an outward period of quiescence, followed by an outburst which relieves the internal strain.

On this theory the irregularity of the intervals and the intensities of the successive outbreaks are not at all surprising. It seems also natural enough that the most violent disturbances should develop the most rapidly, as is observed to be the case.

We shall see later (§ 599) that Hale, after a searching investigation of the phenomena of sun-spots, has adopted a theory which assigns their origin to disturbances in the deeper layers of the sun, well below the photosphere.

240. Many other solar phenomena show, as might be expected, periodic variations which run parallel with those of the spots. The *faculæ*, which are so closely associated with the spots, vary in almost strict proportionality to their numbers (Fig. 94); the number and distribution of the *solar prominences* and also the form and extent of the *corona* (Figs. 96, 97) also show great and regular changes with the sun-spot period; and there is evidence that the total *radiation of heat* (§ 603) from the sun increases with increasing spottedness.

241. Terrestrial Influences of Sun-Spots. There is a conspicuous relation between the abundance of sun-spots and the variations of terrestrial magnetism. The direction and intensity of the earth's magnetic field are never quite constant. In addition to the slow secular variation, which proceeds in the same direction for many years, it is subject to regular *diurnal variations*, and at irregular intervals to sudden and relatively violent disturbances, which are known as *magnetic storms*, and which have been known in extreme instances to change the direction in which the compass points by nearly three degrees in as many minutes. These magnetic storms are frequently accompanied by earth-currents of electricity, which are sometimes strong enough to interfere with the operation of telegraph lines, and by displays of the aurora borealis and aurora australis (which invariably indicate the existence of magnetic disturbance) (§ 658).

It is perhaps worth while to mention that magnetic storms have no connection with the electrical phenomena of thunderstorms, which show no perceptible relation to the sun-spot period.

There is often evidence connecting a magnetic storm with an individual sun-spot (Fig. 98). Maunder finds that during every

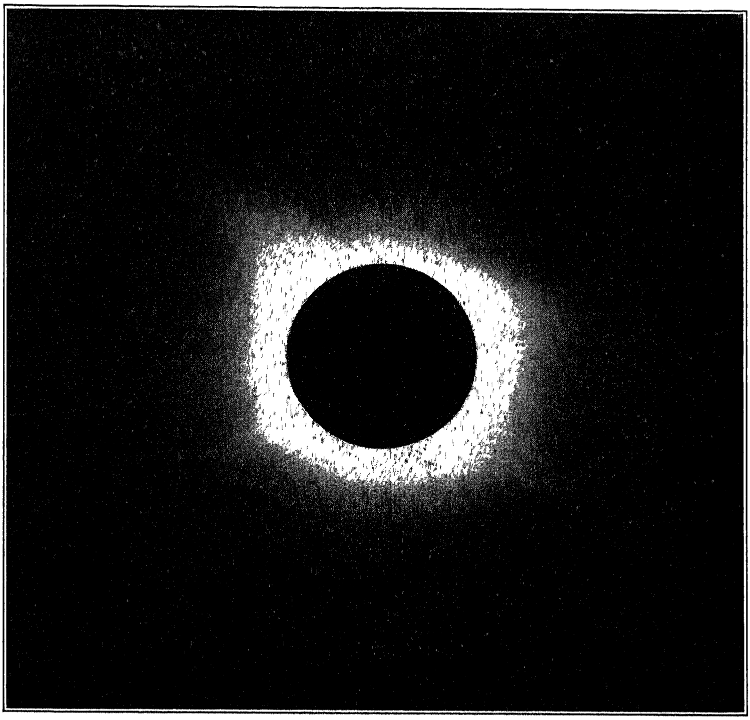


FIG. 97. Eclipse of January 22, 1898, India

At the time of sun-spot minimum the equatorial rays predominate and the short polar rays are sharply defined. Although this eclipse occurred only two years before sun-spot minimum, it still shows some of the characteristics of the maximum type. (From photograph by Lick Observatory)

one of the nineteen great magnetic storms between 1875 and 1903 a large spot, or a spot-group which had been large during a previous rotation of the sun, was on the visible side of the sun and between 20° east and 50° west of the central meridian. On the average these storms occurred twenty-five hours after the corresponding spot had crossed the central meridian. Maunder

numerous, by something between half a degree and one degree, centigrade, for every change of 100 spot-numbers.

There are indications of similar relations between sun-spot changes and the variations in rainfall and other meteorological conditions, but the effects are in all instances so small that they are very hard to separate from the much greater variations, arising from other causes, which happen from year to year.

243. It will be noticed that the accounts of solar phenomena given in this chapter have been for the most part purely descriptive, with a minimum of theoretical explanation. The reason for this is that our knowledge of solar phenomena depends to so great an extent on the information furnished by the spectroscope and other instruments of physical research that it would be premature to attempt to discuss their nature until this evidence has been presented, which will be done in Chapter XV.

EXERCISES

1. If the diameter of the sun were doubled, its density remaining unchanged, what would be the force of gravity at its surface?

2. If the sun were expanded into a homogeneous sphere, with a radius equal to the distance of the earth from the sun, its mass remaining unchanged, what would be the force of gravity at its surface?

Ans 1/1657 of *g*.

3. In this case, what change, if any, would result in the orbit of the earth? *Ans.* None.

4. By how much would a change of $+0''.005$ in the solar parallax alter the accepted value of the astronomical unit? of the diameter of the sun?

5. What are the coördinates (celestial latitude and longitude) of the north pole of the sun?

6. How long would it take a sun-spot on the equator to gain a whole revolution upon one in latitude 30° (§ 225)?

Ans. 532 days (much longer than the life of a spot).

each of the planets, a long, black shadow projecting behind it and traveling with it. Such a shadow is the space from which sunlight is excluded by the intervening body. As the sun, the earth, and the moon are all nearly spherical, and the sun is by far the largest, the shadows of the earth and moon are *cones* with their axes in the line joining the centers of the sun and the shadow-casting body, and with the apex always directed away from the sun. The length of the shadow varies with the distance between the sun and the earth, or the sun and the moon.

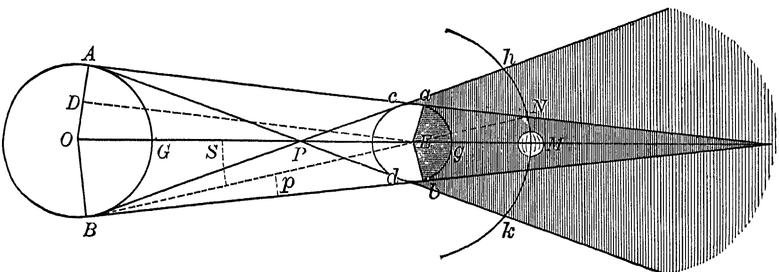


FIG. 99. The Earth's Shadow

246. Dimensions of the Earth's Shadow. The length of the earth's shadow is easily found. In Fig. 99 we have, from the similar triangles OED and ECa,

$$OD : OE = Ea : EC.$$

OD is the difference between the radii of the sun and the earth $= R - r$. $Ea = r$, and OE is the distance of the earth from the sun $= D$. Hence,

$$EC = D\left(\frac{r}{R-r}\right) = \frac{1}{108.1} D.$$

(The number 108.1 is found by simply substituting for R and r their values, R being 109.1 r.) This gives 859,000 miles for the length of the earth's shadow when D has its mean value of 92,870,000 miles. The length varies about 14,000 miles on each side of the mean, in consequence of the variation of the earth's distance from the sun at different times of the year.

From the cone aCb all sunlight is excluded, or would be if it were not for the fact that the atmosphere of the earth, by its refraction, bends some of the rays into this shadow, making it

249. Size of the Earth's Shadow at the Point where the Moon crosses it. Since EC, in Fig. 99, is 859,000 miles, and the distance of the moon from the earth is, on the average, about 239,000 miles, CM must average 620,000 miles, so that MN, the semi-diameter of the shadow at this point, will be 620/859 of the earth's radius. This gives $MN = 2858$ miles, and makes the diameter of the shadow a little over 5700 miles, — about two and two-thirds times the diameter of the moon. It may at times be 200 miles smaller or larger.

An eclipse of the moon, when *central* (that is, when the moon crosses the center of the shadow), may continue *total* for about one hour and forty minutes, the interval from the first contact to

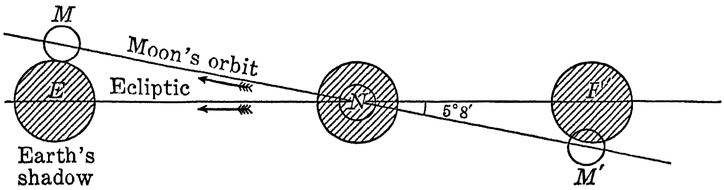


FIG 100. Lunar Ecliptic Limit

If the earth's shadow is at the node N of the moon's orbit when the moon reaches it, there will be a central total eclipse of the moon If the moon overtakes the shadow at E, there will be no eclipse The distance NE is the "lunar ecliptic limit" This distance is greatest (12° 15′) when the apparent diameters of the moon and the earth's shadow have their greatest values

the last being about two hours more. This depends upon the fact that the moon's hourly motion is nearly equal to its own diameter.

The duration of a non-central eclipse varies, of course, according to the part of the shadow traversed by the moon.

250. Lunar Ecliptic Limit. The lunar *ecliptic limit* is the greatest distance of the sun from the node of the moon's orbit at which a lunar eclipse is possible. This limit depends upon the inclination of the moon's orbit, which is somewhat variable, and also upon the distance of the moon from the earth at the time of the eclipse, which is still more variable. Hence we recognize two limits, the major and the minor.

If the distance of the sun from the node, or of the shadow from the opposite node, at the time of full moon exceeds the major limit, an eclipse is impossible; if it is less than the minor, an

Observations of these star occultations made in different parts of the earth furnish data for computing the dimensions of the moon and its parallax, and for determining its position. (2) The study of the heat radiated by the moon during the different phases of an eclipse furnishes information about the absorbing power and temperature of the moon's surface.

252. Computation of a Lunar Eclipse. Since all the phases of a lunar eclipse are seen everywhere at the same absolute instant wherever the moon is above the horizon, it follows that a single computation, giving the Greenwich times of the different phenomena, is all that is needed. Such computations are published in the *Nautical Almanac*. Owing to the uncertainties mentioned above, the time is given only to the nearest tenth of a minute. Each observer has only to correct the predicted time, by simply adding or subtracting his longitude from Greenwich, in order to get the true local time. The computation of a lunar eclipse is not at all complicated.

ECLIPSES OF THE SUN

253. Dimensions of the Moon's Shadow. By the same method as that used for the shadow of the earth (§ 246) we find that the length of the *moon's* shadow at any time is very nearly 1/400 of its distance from the sun, and at new moon *averages* 232,100 miles. It varies not quite 4000 miles each way, ranging from 228,200 to 236,000 miles.

Since the *mean* length of the shadow is less than the mean distance of the moon from the earth (238,900 miles), it is evident that *on the average* the shadow (umbra) will not reach the earth.

On account of the orbital eccentricity, however, the distance of the moon is, for much of the time, considerably less than the mean. The moon may come within 221,700 miles[1] of the earth's center, or about 217,750 miles from its surface. If, at the same time, the shadow happens to have its greatest possible length (that is, if the earth is at aphelion), its point may reach nearly 18,250 miles beyond the earth's surface. In this

[1] This is the minimum distance when the moon is new and at the node. It differs from the absolute minimum distance given in section 193 (p. 165).

cone of the shadow, the sun will be mostly covered by the moon; but if he is near the outer edge of the penumbra, the moon will encroach but slightly on the sun's disk. While, therefore, *total and annular* eclipses are visible as such only by an observer within the narrow path traversed by the shadow spot, the same eclipse will then be visible as a *partial* one everywhere within 2000 miles on each side of that path. The 2000 miles is to be reckoned perpendicularly to the axis of the shadow, and may correspond to a much greater distance on the spherical surface of the earth.

256. Velocity of the Shadow and Duration of Eclipses. If it were not for the earth's rotation the moon's shadow would pass an observer at the rate of nearly 2100 miles an hour, on the average. The earth, however, is rotating, toward the east, in the same general direction as that in which the shadow moves, and at the equator its surface moves at the rate of about 1040 miles an hour. An observer on the earth's equator, therefore, with the moon at its mean distance from the earth and near the zenith, would, on the average, be passed by the shadow with a speed of about 1060 miles an hour (2100 – 1040), or 1600 feet per second. In higher latitudes, where the surface velocity due to the earth's rotation is less, the relative speed of the shadow is higher; and where the shadow falls very obliquely, as it does when an eclipse occurs near sunrise or sunset, the advance of the shadow on the earth's surface may become very swift, — as great as 4000 or 5000 miles an hour.

A *total* eclipse of the sun observed at a station near the equator, under the most favorable conditions possible, may continue total for $7^m\,40^s$. In latitude 45° the duration can barely equal $6\frac{1}{2}^m$. The greatest possible excess of the apparent semidiameter of the moon over that of the sun is only $1'\,19''$.

At the equator an eclipse may continue *annular* for $12^m\,24^s$,[1] the maximum width of the ring of the sun visible around the moon being $1'\,35''$.

In the observation of an eclipse four contacts are recognized: the *first* when the edge of the moon first touches the edge of the sun, the *second* when

[1] This is longer in proportion to the size of the shadow than a total eclipse, because the moon is in apogee and moves more slowly.

and landscape take on strange colors. Animals are perplexed, and birds go to roost. The temperature falls, and sometimes dew appears. A few minutes before the shadow reaches the observer, quivering, ripple-like *shadow bands* appear on every white surface. Just before totality, if the observer is so situated that

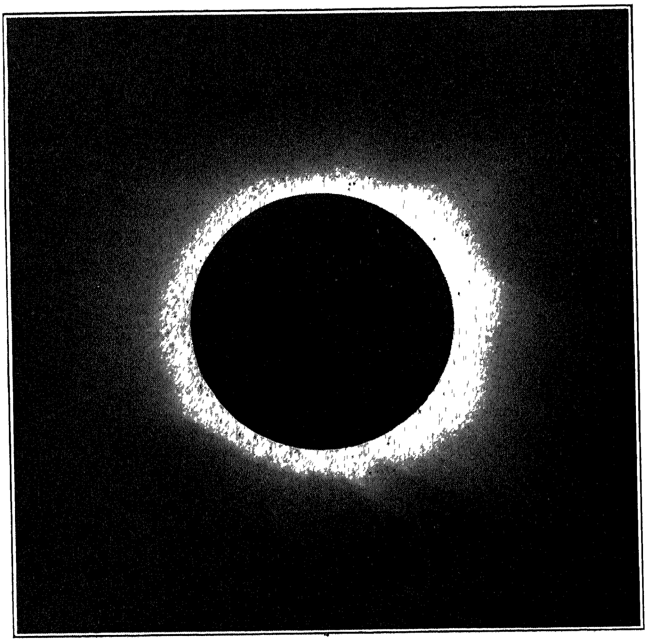


FIG. 104. The Solar Eclipse of January 24, 1925

The corona is of an intermediate type: the long equatorial and the short polar rays are characteristic of the minimum type, whereas the general extension of the corona over the polar regions is typical of the maximum type. (From photograph by Frederick Slocum, Van Vleck Observatory)

his view commands the distant horizon, the moon's shadow is sometimes seen quite distinctly, much like a heavy thunderstorm, and advancing with awe-inspiring swiftness. The last disappearing shred of the sun is often broken up, by the irregularities of the moon's limb, into specks called *Bailey's beads*. With the arrival of the shadow the *corona, chromosphere,* and *prominences* become visible, together with the brighter planets and the stars of the first two or three magnitudes. The sud-

In another minute the inner corona also is drowned out by the returning light and the spectacle is over.

259. Observation of an Eclipse. The professional astronomer is too busy during the few minutes of totality to enjoy the beauties of this grandest of all spectacles. He and his assistants have spent months of preparation in constructing apparatus, in traveling perhaps to some distant and primitive region, in setting up and adjusting the instruments, and in rehearsing time and again the program which must be carried out without a hitch. During totality many plates are exposed and changed, leaving time for the astronomer to indulge in but a hurried glance at the corona.

A total eclipse of the sun offers opportunities for numerous observations of great importance which are possible at no other time. We mention:

(1) Observations for the determination of the relative positions of the sun and moon. These include the times of the four contacts (§ 256), the geographical boundaries of the shadow zone, and photographs of the partial phases.

(2) Photographic observation of the sky near the sun, including the search for possible intra-Mercurial planets (§ 423), which has practically proved their absence, and measures of the deflection of the light from stars seen near the sun in the sky, which provides one of the tests of the theory of general relativity (§ 363).

(3) Direct photography of the corona and prominences.

(4) Spectroscopic observation of reversing layer, chromosphere, and corona.

(5) Photometric measurement of the intensity of the corona and of the partial phases.

(6) Bolometric measurement of the heat radiation of the corona.

(7) Observation of the degree of polarization of the light of the corona.

(8) Miscellaneous observations upon the shadow bands, the meteorological changes during the progress of the eclipse (barometric pressure, temperature, wind, etc.), and of the effects upon the magnetic elements and upon the transmission of radio.

sun being too far from the node at all four of the full moons which occur nearest to the time of its node passage.

In a *calendar* year (of $365\frac{1}{4}$ days) it is, however, possible to have *three* lunar eclipses. If one of the moon's nodes is passed by the sun in January, it will be reached again in December, the other node having been passed in the latter part of June, and there may be a lunar eclipse at or near each of these three node passages. This actually occurred in 1898 and 1917, and will happen again in 1982.

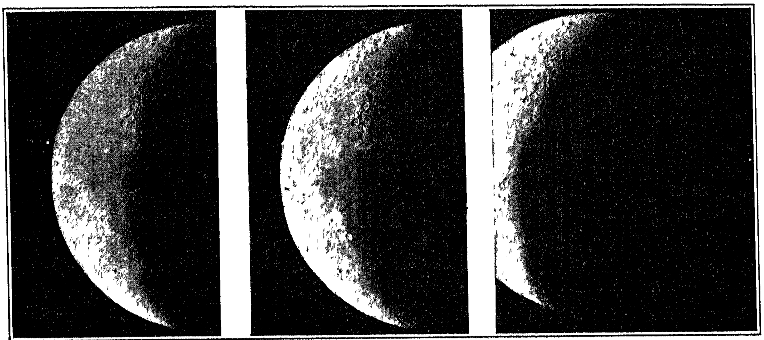
As to solar eclipses, it is sufficient to say that the solar ecliptic limits are so much larger than the lunar that there *must be at least one solar* eclipse at each node passage of the year, at the new moon, and that there may be *two*, one before and one after, thus making four in the eclipse year. (When there are two solar eclipses at the same node, there will always be a total lunar eclipse at the full moon between them.) In the *calendar* year a fifth solar eclipse may come in if the first eclipse month falls in January. Since a year with five solar eclipses in it is sure to have two lunar eclipses in addition, they will make up seven in the calendar year. This will happen next in 1935; and in 1917 there were also seven eclipses, — four of the sun and three of the moon.

262. Relative Frequency of Solar and Lunar Eclipses. Taking the whole earth into account, the solar eclipses are the more numerous, nearly in the ratio of *three to two*. *It is not so, however, with those which are visible at a given place.* A solar eclipse can be seen only from a limited portion of the globe, while a lunar eclipse is visible over considerably more than half the earth (either at the beginning or the end, if not throughout its whole duration), and this more than reverses the proportion between the lunar and solar eclipses for any given station.

Solar eclipses that are *total* somewhere or other on the earth's surface are not very rare, averaging one for about every year and a half. But *at any given place* the case is very different. Since the track of a solar eclipse is a very narrow path over the earth's surface, averaging only 60 or 70 miles in width, we find that in the long run a total eclipse happens at any given station only once in about 360 years.

is very small. Observations of occultations determine the place of the moon in the sky with great accuracy, and when made at a number of widely separated stations they furnish a precise determination of the moon's parallax and also of the difference of longitude between the stations.

Occasionally the star, instead of disappearing suddenly when struck by the moon's limb (faintly visible by earth-shine), appears to cling to the limb for a second or two before vanishing. In a few instances it has been reported as having reappeared and disappeared a second time, as if it had been for a moment visible through a rift in the moon's crust. In some cases



C *B* *A*

FIG. 106. Occultation of Aldebaran, March 22, 1904

A, Aldebaran is seen at the right-hand border just before disappearance at the dark limb of the moon. *B*, just emerging at the bright limb *C*, $1^m\ 40^s$ after emergence. The exposure time was correct for the moon; the image of the bright star is enlarged by overexposure. (From photograph by R. J Wallace, Yerkes Observatory)

the anomalous phenomena have been explained by the subsequent discovery that the star was double, but many of them still remain mysterious; it is quite likely that they were often illusions of the observer, due to physiological causes.

265. Recent and Coming Eclipses. Total eclipses visible in the United States occurred on May 28, 1900, from Louisiana to Virginia, duration of totality $1\frac{1}{2}$ minutes; on June 8, 1918, from the state of Washington to Florida, 2 minutes; on September 10, 1923, California, $3\frac{1}{2}$ minutes; and on January 24, 1925, from Minnesota to Connecticut, 2 minutes. The third of these was obscured by unexpected bad weather, although observations were secured in Mexico. The weather was fair for the second, and

Eclipses will occur on June 29, 1927, England and Norway, 30 seconds, on May 9, 1929, Sumatra, 5 minutes; on August 31, 1932, Vermont to Maine, $1\frac{1}{2}$ minutes; on June 8, 1937, Pacific Ocean, 7 minutes. It will be noticed that the three eclipses of longest duration are repetitions at intervals of the Saros.

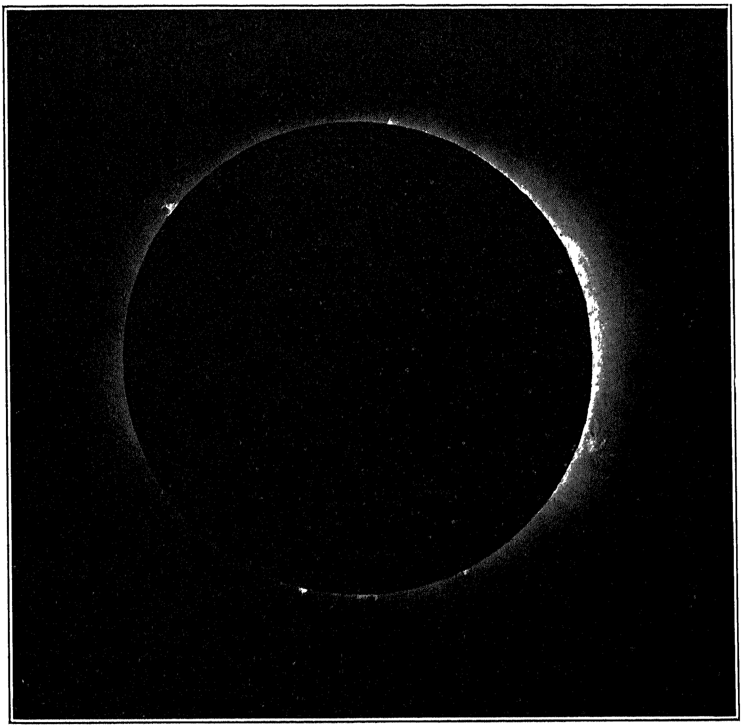


FIG. 107 *B*. The Eclipse of June 8, 1918, Goldendale, Washington

This photograph was taken a few seconds before third contact. Comparison with Fig 107 *A* shows the motion of the moon On the western limb the chromosphere is appearing and the huge prominences are entirely uncovered Note the coronal arches over one of these prominences (From photograph by Lick Observatory)

Computations made in the office of the *American Ephemeris*, and communicated before publication by courtesy of the superintendent of the Naval Observatory and the director of the *Nautical Almanac*, show that the solar eclipse of April 28, 1930, will be just total. The central line runs from California through Nevada to Montana, and the maximum duration of

CHAPTER IX

THE PLANETS IN GENERAL

NAMES, DISTANCES, AND PERIODS · APPARENT MOTIONS · THE ELEMENTS OF A PLANET'S ORBIT · THE DETERMINATION OF ORBITS · STUDY OF THE PLANETS THEMSELVES: DIAMETER, MASS, SURFACE PECULIARITIES, ROTATION · SATELLITE SYSTEMS · CLASSIFICATION OF THE PLANETS

266. The *stars* preserve their relative configurations, however much they may alter their positions in the sky from hour to hour. The Dipper always remains a dipper in every part of the diurnal circuit.

But certain of the heavenly bodies, and the most conspicuous of them, behave differently. The sun and the moon move always steadily eastward through the constellations; and a few others, which look like brilliant stars, but are not stars at all, creep back and forth among the star groups in a less simple manner.

These moving bodies were called by the Greeks *planets*, that is, wanderers. The steadiness of their light, that is, the absence of twinkling (§ 117), unless they are low down near the horizon, also distinguishes them from the stars. The Greeks enumerated seven, — Mercury, Venus, Mars, Jupiter, and Saturn, and, in addition, the Sun and Moon.

267. List of Planets. At present the sun and moon are not reckoned as planets, but the earth is, and the number known to the ancients has been increased by two new worlds (Uranus and Neptune, of great magnitude though inconspicuous on account of their distance), besides a host of little bodies called asteroids.

The list of the principal planets in their order of distance from the sun stands thus at present: Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and Neptune.

Between Mars and Jupiter, where there is a wide gap in which another planet might be expected, there have been discovered more than a thousand asteroids, which may represent a single planet that was somehow "spoiled in the making," so to speak.

The *sidereal period* of a planet is the time of its revolution around the sun, from a *star* to the same star again, *as seen from the sun.* The *synodic period* is the interval between two successive times when the planet occupies the same position in relation to the sun *as seen from the earth.*

Let us imagine ourselves on the sun, watching the planets traveling around their orbits at different speeds. The earth and another planet start together. One gains a lap on the other and completes the synodic period of the planet. Mercury travels so fast that it catches up with the earth again before the earth has gone more than one third of the way around. Neptune moves only about two degrees while the earth is making a complete circuit, and is overtaken in $367\frac{1}{2}$ days. The earth and Mars are so evenly matched that it takes the earth over two years to overtake Mars.

The sidereal and synodic periods are connected by the same relation as the sidereal and synodic months (§ 186), namely, $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$, in which E, P, and S are, respectively, the sidereal periods of the earth and the planet, and the planet's synodic period; and the numerical difference between $\frac{1}{P}$ and $\frac{1}{E}$ is to be taken *without regard to sign*; that is, for an *inferior* planet, $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$; for a *superior* one, $\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$.

269. The Harmonic Law. There is evidently a relation between the distance of a planet and its period; the more distant planets have the longer periods. The exact nature of this relation was discovered by Kepler early in the seventeenth century, and constitutes his famous harmonic law, *the squares of the periods are proportional to the cubes of the mean distances from the sun*, which may readily be verified from the table. This relation was shown by Newton to be a consequence of the law of gravitation, and will be discussed in Chapter X.

There is a curious approximate relation between the distances of the planets from the sun, usually known as *Bode's law* because first brought prominently into notice by Bode in 1772, though it appears to have been discovered by Titius of Wittenberg some years earlier.

physical explanation which may be understood in the future. For Neptune the law breaks down, and it really does so for Mercury, for which the law should give the distance 5.5 (adding half of 3, instead of 0, to 4).

270. Planetary Configurations. Fig. 109 illustrates the meaning of the terms used in describing the position of a planet with respect to the sun. E is the position of the earth, the inner circle is the orbit of an *inferior* planet (Mercury or Venus), and the outer circle is that of a *superior* planet, — Mars, for instance.

The *elongation* of a planet is the angle between lines drawn from the observer to the planet and to the sun, that is, the apparent angular distance of the planet from the sun; for a planet at P it is the angle SEP.

For a *superior* planet the elongation can have any value from 0° to 180°. For an *inferior* planet there is a certain maximum value, called the *greatest elongation*, which must be less than 90°. This greatest elongation is the angle between a line drawn from the earth to the sun and another line drawn tangent to the planet's orbit, — the angle SEV in the figure.

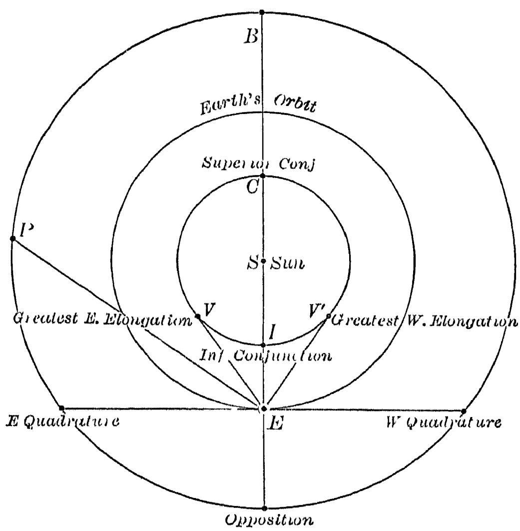


Fig. 109. Planetary Configurations

Absolute conjunction occurs when the elongation of the planet is zero; *superior conjunction*, when the planet is in line with and beyond the sun; *inferior conjunction*, when the planet is between the earth and the sun, — a position which is impossible for a superior planet. *Conjunction in longitude* occurs when the planet's longitude is the same as the sun's; *conjunction in right ascension*, when it has the same right ascension as the sun.

form and size, and in the same plane, always keeping its motion opposed to our own though going around this apparent orbit *in the same direction* as the earth (just as any two opposite points on the circumference of a revolving wheel are always moving in opposite directions though going the same way around the axis). And since the planets are really revolving around the sun, it follows that their apparent, or *geocentric*, motion is a combination of two motions, — that of a body moving once a year around the circumference of a circle[1] equal to the earth's orbit, while at the same time the center of that circle is carried around the sun in the real orbit of the planet, and in the planet's period. Jupiter, for instance, as seen from the earth, appears to move as in Fig. 110.

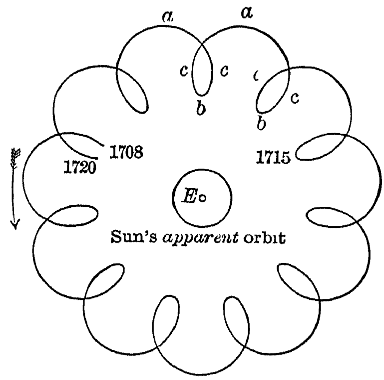


FIG. 110. Geocentric Motion of Jupiter from 1708 to 1720

After Cassini

This is the orbit that we should find if we were to attempt to map it out by the method used for determining the form of the orbit of the earth around the sun (§ 158), that is, by observing the *direction* of the planet from the earth and at the same time measuring its *angular diameter* in order to get its relative distances at different times. Practically, however, this method would not here succeed very well, since the planet's apparent diameter is too small to permit the necessary precision in determining the variations of distance.

A motion of the kind represented in the figure is loosely called epicycloidal, — not quite accurately, because the orbits concerned are not true circles, so that the loops are of varying size.

The Ptolemaic theory of the solar system was fundamentally an acceptance of this apparent motion of the planets, relative to the earth, as real, though the theory involved in addition certain serious errors of arrangement and proportion (§ 277).

[1] The "circles" spoken of here are, strictly, ellipses of small eccentricity.

Fig. 111 shows the loops made by Jupiter and Saturn in 1901, when they were very near each other and, for a short time, near the rapidly moving Venus.

Venus may attain a latitude of almost 9°, Mars nearly 7°, and Mercury almost 5°. None of the other planets can reach 3°.

Certain of the asteroids have orbits greatly inclined to the ecliptic and very eccentric. The description of apparent motions as given above would therefore require very serious modification in their case. Eros is sometimes found in circumpolar regions more than 40° north of the ecliptic, sometimes its nearest approach to the earth does not coincide with the time of its opposition within several weeks, and sometimes at the time of its opposition its motion is more nearly from *north to south* than from east to west Fig. 112 shows the track which Eros will follow through opposition in 1931

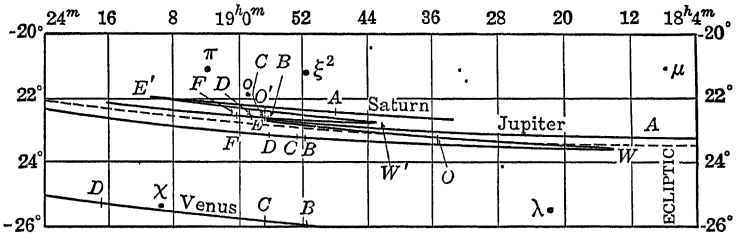


Fig. 111 Apparent Motions of Saturn, Jupiter, and Venus in 1901

Saturn and Jupiter were describing their loops through opposition, while Venus passed close by to the south The opposition points, at the middle of the retrograde arcs, are marked O; the stationary points, E and W (east and west) Corresponding points A, B, C, D, and F are reached by two or more planets on the same date. Venus and Jupiter are in conjunction at B, Venus and Saturn at C, Jupiter and Saturn at F. The dates are as follows Saturn, E', April 25; O', July 5, W', September 14. Jupiter, E, April 29; O, June 30, W, August 30. A, January 30 0, B, November 17 7, C, November 18 7; D November 23 0; F, November 27 7 Saturn, being more distant, describes a shorter loop than Jupiter and moves more slowly Jupiter passed the descending node near the time of opposition, and Saturn was approaching the corresponding point in its orbit

275. Motion of the Planets in Elongation, that is, with Respect to the Sun's Place in the Sky. The visibility of a planet depends mainly on its *elongation*, because when near the sun the planet will be above the horizon only by day. Considered from this point of view, there is a marked difference between the inferior planets and the superior planets.

(1) *The superior planets always drop steadily westward with respect to the sun's place in the heavens*, continually increasing their western elongation or decreasing their eastern elongation.

ever, it reappears in the morning, rising before the sun as a morning star, and passes on to western quadrature, when it rises near midnight. Thence it moves on to opposition, when it is nearest and brightest, and rises at sunset. Still dropping westward, and now receding, it passes to eastern quadrature and is on the meridian at sunset. Thence it still crawls sluggishly westward as an evening star until it is lost in the twilight and completes its synodic period by again reaching conjunction.

276. (2) The *inferior planets*, on the other hand, apparently *oscillate* across the sun, moving out equal or nearly equal distances on each side of it, but making the westward swing, between us and the sun, much more quickly than the eastward swing, through superior conjunction.

At superior conjunction an inferior planet is moving eastward *faster* than the sun. Accordingly it creeps out into the twilight as an evening star and continues to increase its apparent distance from the sun until it reaches its *greatest eastern elongation* (47° for Venus; from 18° to 28° for Mercury). Then the sun begins to gain upon it, and as the planet itself soon begins to retrograde, the elongation diminishes rapidly and the planet hurries back to *inferior conjunction*, passes it, and then, as a morning star, moves swiftly out to its western elongation. There it turns and climbs slowly back to superior conjunction again.

277. The Ptolemaic System. Assuming the fixity and central position of the earth and the actual revolution of the heavens, Ptolemy (who flourished at Alexandria about A.D. 140) worked out the system which bears his name.

In his great work, the *Almagest* (from Arabic *al*, "the," and the Greek μεγίστη meaning "greatest"), which for fourteen centuries was the authoritative "scripture of astronomy," he showed that all the apparent motions of the planets (including the sun and moon) so far as then observed, could be accounted for by supposing each planet to move around the circumference of a circle called the epicycle, while the center of this circle, sometimes called the fictitious planet, itself moved *around the earth* on the circumference of another and larger circle called the deferent (see Fig. 113).

It was as if the real planet were carried on the end of an arm which turned around the fictitious planet as a center in such a way as to point toward or from the earth at times when the planet was in line with the sun.

ent diurnal revolution of the stars. He also showed that nearly all the known motions of the planets could be accounted for by supposing them to revolve around the sun, with the earth as one of them, in orbits *circular* but slightly *out of center*. His system, as he left it, was nearly that which is accepted today. He was, however, obliged to retain a few small epicycles to account for certain of the irregularities.

Up to this time no one dared to doubt the exact circularity of celestial orbits. It was considered metaphysically improper that heavenly bodies should move in any but *perfect* curves, and the circle was regarded as the only perfect one. It was left for Kepler, some sixty years later than Copernicus, to show that the planetary orbits are *elliptical*, and to bring the system into substantially the form in which we know it now.

It was nearly a century before the Copernican system, with the improvements of Kepler, finally replaced the Ptolemaic. In our oldest American universities, Harvard and Yale, the Ptolemaic was for a considerable time taught in connection with the Copernican.

279. Tychonic System. Tycho Brahe, who came between Copernicus and Kepler, found himself unable to accept the Copernican system for two reasons One was that it was unfavorably regarded by the Church, and he was a good churchman; the other was the really scientific objection that if the earth moved around the sun, the fixed stars all ought to appear to move in a corresponding manner, each star describing annually an ellipse in the heavens of the same apparent dimensions as the earth's orbit seen from the star Technically speaking, each star ought to have an *annual parallax*.

His instruments were by far the most accurate that had ever been made, and he could detect no such parallax (although it really existed and can now be observed); hence he concluded, not illogically though incorrectly, that the earth must be at rest.

He rejected the Copernican system, placed the earth at the center of the universe, according to the then received interpretation of Scripture, made the sun revolve around the earth once a year, and then (this was the peculiarity of his system) made the apparent orbit of the sun the *common deferent* for the epicycles of all the other planets, making them revolve around the sun.

This theory just as fully accounts for all the motions of the planets as the Copernican or Ptolemaic, but, like the Ptolemaic, breaks down absolutely when it encounters the *aberration of light* and the *annual parallax of the stars*,

node. This angle lies in the plane of the ecliptic and is measured from the vernal equinox in the direction of the earth's motion. The planet passes from the lower, or southern, side of the plane of the ecliptic to the northern at the point n in its orbit, so that N is the ascending and N' the descending node.

The fifth and last of the elements which belong strictly to the orbit itself is ω, which defines the direction in which the major axis of the ellipse (the line pA) lies in the plane $ORBT$. It is

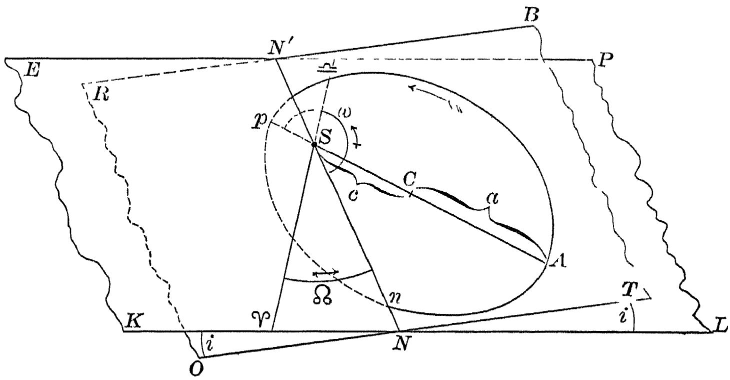


FIG. 114. The Elements of a Planet's Orbit

measured in the plane of the planet's orbit and in the direction of the planet's motion. The "longitude of the perihelion," π, is given by the equation $\pi = \Omega + \omega$.

When the motion of the body in its orbit is retrograde (that is, in the opposite sense from that of the earth), the inclination is regarded as greater than 90°. Thus, a comet moving backward in a plane making an angle of 10° with the plane of the ecliptic, is said to have an inclination of 170°. The definitions of Ω and ω then apply without alteration.

If we regard the orbit as an elliptical wire hoop suspended in space, these five elements completely define its *position*, *form*, and *size*. The *plane* of the orbit is fixed by the two elements i and Ω ; the *position* of the orbit in this plane, by ω; the *form* of the orbit, by e; and, finally, its *size*, by a.

To determine where the planet will be at any date we need two more elements:

We must distinguish here between the *true* direction of a planet from the earth at any moment and the *apparent* direction. An ephemeris of true directions is computed immediately from the elements of the planet's orbit. The apparent direction at the moment T differs from the true direction, both because of the aberrational displacement resulting from the earth's motion (§ 162) and because of the so-called *planetary aberration*. This is the angular distance through which the planet moves relatively to the earth in the time, t, that it takes light to travel from the planet to the earth. At the moment T the observer sees the planet where it was at the time $T - t$, when the light, by which it is observed, left it Both aberrations may be allowed for, either by applying a correction to the true position at the time T (the procedure followed in the nautical almanacs) or (as readily can be proved) by antedating the observation by the light-time t before comparing with the ephemeris (Fig. **115**). In dealing with observations of asteroids or comets great care must be taken to find out whether this correction has been applied, and if so, how.

THE DETERMINATION OF ORBITS

283. The Modern Method. By utilizing to the full the knowledge that can be obtained from the theory of gravitation it is possible to calculate all the elements of a planet's orbit from three accurate observations of its right ascension and declination, separated by a few weeks (in a few special cases a fourth observation is necessary).

The observations may be made with the meridian circle, with a filar micrometer, or by photography.

The theory is intricate and the calculation long and complicated (§ 319), but a skilled computer can carry the work through in a day or two. This method is habitually employed by astronomers when a new asteroid or comet is discovered. This problem was solved in 1801 by Gauss, then a young man of twenty-three, in connection with the discovery of Ceres, the first of the asteroids, which, after its discovery by Piazzi, was lost to observation by passing into conjunction with the sun.

284. Older Geometrical Methods. In earlier times, however, when gravitational theory was unknown, the orbits of the planets

tions, but the synodic period can, for it is the interval between successive conjunctions or oppositions.

The exact instant of opposition is found from a series of right ascensions and declinations observed about the proper date. By comparing the deduced longitudes of the planet with the corresponding longitudes of the sun we easily find the precise moment when the difference was 180°. When the synodic period is found, the sidereal is at once given by the equations in section 268 (p. 235), namely, $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$ for an inferior planet, and $\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$ for a superior one. In the first case $P = \frac{S \times E}{S + E}$.

It will not answer for this purpose to deduce the synodic period from two *successive* oppositions, because, on account of the eccentricity of the orbits, both of the planet and of the earth, the synodic periods are notably variable. The observations must be sufficiently separated in time to give a good determination of the *mean* synodic period.

In the case of all the older planets we have observations running back nearly two thousand years, so that no difficulty arises on this score.

286. The position of a planet (for example, Mars) in its orbit may then be found as follows: Let A (Fig. 117) be the position of the earth at any date when Mars was observed and found to lie on the line AM. After one sidereal period (686 95 days) the earth will be at C. Even if no observation was obtained on this date, the direction of the line CM, on which the planet then lay, can be found by interpolation among neighboring observations. The intersection M of these two lines is the planet's position at both dates. What could thus be done graphically could be done more accurately by trigonometric calculation.

Two observations of Venus, V, separated by a period of 225 days, will similarly mark out a point on its orbit.

In practice the lines AM and CM will usually not be in the plane of the ecliptic, but by considering the planet's longitudes alone the construction just given will fix the projection of the planet on the plane of the ecliptic. From the observed latitudes the distance above or below this plane may then be found.

cal unit, and a map of the solar system can be formed, correct in all its proportions but without a scale of miles. The measurement by trigonometrical parallax of any one planetary distance in the solar system will then suffice to express them all in miles. The parallax of the sun cannot be measured directly with accuracy; the sun is too bright and too large, and its parallax is small. The asteroid Eros has proved to be the most favorable subject for parallax measurements. It can be as accurately observed as a star, and at opposition may come within little more than thirteen million miles of the earth.

STUDY OF THE PLANETS THEMSELVES

In discussing the individual characteristics of the planets we have to consider a variety of different data, obtained by telescopic observation, — micrometric, spectroscopic, photometric, and radiometric measurements, — and then to study their *diameters*, their *masses* and *densities*, their *satellite systems*, their *axial rotation*, their *surface markings*, their *light* and *temperature*, their *atmospheres* if present, and the *physical conditions* which prevail on their surfaces.

288. Determination of Size: Diameter, Surface, and Volume. The size of a planet is found by measuring its *apparent* diameter in seconds of arc with some form of micrometer (§ 82) attached to a powerful telescope. Since from the elements of the orbit of a planet and of the earth we can find the distance of the planet from the earth at any time in astronomical units, we can at once deduce the real linear diameter from the apparent diameter D'', by the equation given in section 109 (p. 96), namely,

$$\text{linear diameter} = \Delta \sin D'', \text{ or } \frac{\Delta D''}{206{,}265},$$

Δ being the distance of the planet from the earth. This will give the linear diameter as a fraction of the astronomical unit and can be converted into miles by simply multiplying it by 92,870,000, the number of miles in the unit, or into kilometers by multiplying by 1.4945×10^8.

For many purposes it is convenient to express the planet's radius in terms of the earth's radius by dividing half the diam-

In the case of Mercury the mass is still very uncertain. Venus, however, disturbs the earth and Mars sufficiently to give a good determination of her mass.

290. Surface Gravity and Density. When the mass has been determined, the surface gravity and density follow at once. The surface gravity is P/r^2 times that on the earth. The *density*, compared with the earth, is simply P/r^3; if we want the density as compared with water, we must multiply the result by 5.52, the density of the earth. Any error in the measured diameter, of course, very seriously affects the computed density and gravity.

291. Rotation Period and Data connected with it. The length of the planet's "day," when it can be determined at all, is usually ascertained by observing some well-marked spot on its disk and noting the times of its successive returns. An approximate value of the rotation period is obtained from the observation of such returns during a few days or weeks, and this is afterward corrected by data furnished from observations extending over the longest interval obtainable, a century or more if possible.

Mars, however, is the only planet of which the rotation period is known with great accuracy; the others either show no well-defined markings or show only such markings as seem to be more or less movable on the planet's surface, like spots on the sun.

In reducing the observations, account has to be taken of the continual change in the direction of the planet from the earth and also of the variations of its distance, which alter the time taken by light to reach us.

Even when the planet has no distinct surface markings, methods are available which may yield at least an approximate value for the period of rotation. (1) The speed of approach and recession of opposite limbs may be measured with the spectroscope. (2) Periodic variation of brightness sometimes arises from the presentation, by rotation, of areas of various reflecting power. (3) The oblateness of the planetary disk may, with certain assumptions concerning the internal distribution of the planet's mass, give a measure of the centrifugal force and hence of the speed of rotation.

The *inclination* of the planet's equator to the plane of its orbit, and the positions of its poles and equinoxes, are deduced

sible to identify such as are permanent and to chart them more or less perfectly. Photography is proving an increasingly valuable aid but cannot compete with visual observation in the study of the finest details.

Just as the study of the surface of the earth is known as geography, so that of the surface of the moon is called selenography, and that of the surface of Mars, areography.

294. The Satellite Systems. The principal data to be determined in respect to these systems are the distances and periods of the satellites. These are found, along with the eccentricities, inclinations, etc., by *micrometric* measures of the apparent distance and direction of each satellite from the planet, or from other satellites. The latter is now the usual method, since the distance and direction between two satellites (which appear as mere points of light) can be measured much more precisely than between a satellite and the center of the large disk of a planet. The reduction of the observations in this case is, however, very complicated.

The diameters of some of the larger satellites are measurable. Those of the smaller ones can be only roughly guessed at, on the basis of their photometrically observed brightness. In several instances satellites show periodic variations in brightness, which indicate that they make an axial rotation in the time of one revolution around the primary, just as our moon does.

Where there are a number of satellites attending a planet, their mutual perturbations furnish a very interesting subject of study and make it possible to determine their *masses* relative to that of the planet.

All those satellites of the planetary system which lie relatively near their primary (at a distance less than ten or twelve times its diameter) move in very nearly circular orbits, whose planes are nearly coincident with that of the equator of the primary. The remoter satellites, among which the moon is to be counted, usually have orbits of considerable eccentricity and inclination.

295. Classification of Planets. Humboldt has classified the planets into two groups, — the terrestrial planets, as he calls them, and the major planets. The terrestrial group contains the four planets nearest the sun, — Mercury, Venus, Earth, and

GENERAL PRINCIPLES

298. The laws of motion, formulated by Newton (though some of the principles involved had been recognized earlier by Galileo), form the basis of this theory. According to the first law *a moving body on which no force is acting travels in a straight line with uniform speed.* Failure to realize this held back the advance of mechanical science for nearly two thousand years after the time of Aristotle, while it was supposed that rest was more natural to a body than motion. This is not true; mere motion implies no acting force. According to Newton, *change of motion* only, either in speed or in direction, implies such action. The second law states that *change of motion is proportional to the force which acts on a body (the impressed force), and takes place in the direction of this force.*

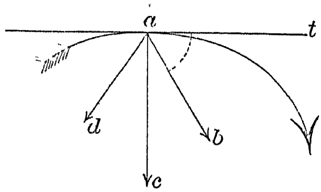


FIG 119. Curvature of an Orbit

Thus, if the force is directed forward, the body will move faster and faster, traveling in the same straight line; if it is directed backward, the body will be retarded, still moving in the same line; if it is directed laterally, the path will curve toward the side to which the force urges it. Conversely, if the speed of a body moving in a curve (Fig 119) increases, we know that the acting force pulls not only crosswise but forward, as *ab*; if it decreases, the force is directed backward, like *ad*; if the speed is constant, the force is exactly crosswise, along *ac*. This force may be, and often is, the resultant of several forces, but they act as one.

By "motion" in this law Newton meant what is now called *momentum*, the product of the velocity of the moving particle by its mass; and by "change" he meant the *rate of change* per unit of time. The rate of change of velocity alone, whether in amount or in direction, is called *acceleration*. If m is the mass, a the acceleration, and f the force, then

$$f = ma.$$

The third law states that *action and reaction are equal and opposite.* If, for example, the earth attracts the moon, the moon

301. Linear, Angular, and Areal Velocities. The *linear velocity* of a particle is the number of linear units (centimeters, feet, miles) that it moves over in a unit of time, say a second. Its symbol is usually V. The *angular velocity* is the number of units of angle (radians or degrees) swept over by the radius vector in a unit of time. The usual symbol for this is ω. The *areal velocity* is the area swept out by the radius vector in unit time (square miles per second) and is constant if the force is central.

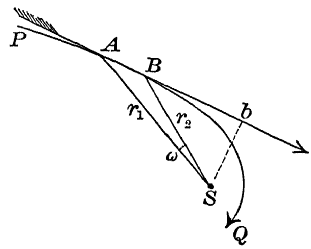


FIG. 121. Linear and Angular Velocities

In Fig. 121, if AB is the length of the path described in unit time, AB is the *linear velocity* V, the angle ASB is the *angular velocity* ω, and the area ASB is the *areal velocity*, which is constant. Calling this A and regarding the sector as a triangle (which it is nearly enough), we have $A = \frac{1}{2} V \times x$, x being the line Sb drawn from the center of force perpendicular to the line of motion; so that if we regard AB as the base of the triangle, x is its altitude. Hence we have the equation

$$V = \frac{2A}{x}. \tag{1}$$

Also, $A = \frac{1}{2} r_1 r_2 \sin ASB$. Since in a second of time the angle ASB, or ω, is so small that it may be taken equal to its sine, and $r_1 r_2$ equals (sensibly) r^2, we have

$$\omega = \frac{2A}{r^2}. \tag{2}$$

In every case of motion under a central force, therefore, (1) the *areal velocity is constant* in all parts of the orbit; (2) the *linear velocity varies inversely as the perpendicular drawn from the center to the line of motion*; and (3) the *angular velocity varies inversely as the square of the radius vector.*

These three statements are not independent laws, but simply different geometrical equivalents for one law. They hold good regardless of the nature of the force, requiring only that it act *directly toward, or from, the center*, along the line of the radius vector.

As the orbits of the principal planets are all nearly circular, these formulæ will find frequent application.

304. Kepler's Laws. Early in the seventeenth century Kepler discovered, as unexplained facts, three laws which describe the motions of the planets, — laws which still bear his name He worked them out from a study of the observations which Tycho Brahe had made during many preceding years upon the planets, especially Mars. They are as follows:

(1) The orbit of each planet *is an ellipse with the sun in one of its foci* (§ 311).

(2) *The radius vector of each planet describes equal areas in equal times.*

(3) *The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun; that is,* $t_1^2 : t_2^2 :: a_1^3 : a_2^3$. This is the so-called harmonic law.

To make sure that the student apprehends the meaning and scope of this third law we add a few simple examples of its application.

1. What would be the period of a planet having a mean distance from the sun of one hundred astronomical units, that is, a distance a hundred times that of the earth?

$$1^3 \cdot 100^3 = 1^2 \text{ (year)} . X^2,$$

whence
$$X \text{ (in years)} = \sqrt{100^3} = 1000 \text{ years}$$

2 What would be the distance from the sun of a planet having a period of 125 years?

$$1^2\text{(year)} \cdot 125^2 = 1^3 \cdot X^3,$$

whence
$$X = \sqrt[3]{125^2} = 25 \text{ astronomical units.}$$

3. What would be the period of a satellite revolving close to the earth's surface?

$$\text{(moon's distance)}^3 \cdot \text{(distance of satellite)}^3 = (27.3 \text{ days})^2 \ X^2,$$

or,
$$60^3 : 1^3 = 27\ 3^2 \cdot X^2,$$

whence
$$X = \frac{27\ 3}{\sqrt{60^3}} = 1^{\text{h}}\ 24^{\text{m}}$$

305. Inferences from Kepler's Laws. From what has already been said (§ 300), Kepler's second law indicates that *the force which determines the orbits of the planet is directed toward the sun.*

This, by itself, tells us nothing about the magnitude of the force; but from the fact that the orbit is an ellipse, with the sun at its focus, Newton proved (by a demonstration a little

308. The constant of gravitation (G in the above equation) is believed to be an absolute constant of nature, like the velocity of light, — the same throughout the material universe, although the numerical value which is assigned to it depends upon the units used in measuring mass, length, and time. In the c.g.s. (centimeter-gram-second) system its value is 6.673×10^{-8}. Two masses of one gram, one centimeter apart, therefore attract one another with a force of 6.67×10^{-8} dynes, or about one fifteen-millionth of a dyne.

A dyne (the c.g.s. unit of force) is that force which, acting for one second on a mass of one gram, imparts to it a velocity of one centimeter per second; it is about equal to the weight of one milligram. Gravitational attraction between bodies of ordinary dimensions is thus exceedingly small, and very delicate apparatus must be employed to measure it (§ 148). Nevertheless, the constant G has been determined to about one part in a thousand. It is only because of the huge masses of the sun and the planets that their gravitational attraction becomes important.

309. Newton's Test of his Theory of Gravitation by the Motion of the Moon. When, in 1665, Newton first conceived the idea of universal gravitation, he saw at once that the moon's motion around the earth ought to furnish a test. Since the moon's distance (as was known even then) is about sixty times the radius of the earth, the distance it should fall toward the earth in a second ought to be, if his idea of gravitation was correct, 1/3600 of 193 inches (the distance which a body falls in a second at the earth's surface. This assumes, however, that the earth attracts as if its mass were all collected at its center — a theorem Newton had much trouble in proving, as it involved the use of his new mathematical invention of "fluxions").

Now 1/3600 of 193 inches is 0.0535 inches. Does the moon fall toward the earth, that is, deflect from a straight line, by this amount each second?

According to the law of central forces, considering the moon's orbit as circular, its acceleration is

$$a = 4\,\pi^2 \times \frac{r}{t^2},$$

closed curve, returning into itself, and such that the *sum* of the distances of any point N of the curve from the two foci equals the major axis, that is, $FN + F'N = PA$.

The hyperbola does not return into itself The portions PN'' and Pn'' of the curve go off indefinitely, becoming ultimately nearly straight, and diverging from each other at a definite angle. In the hyperbola the *difference* of the distances of a point

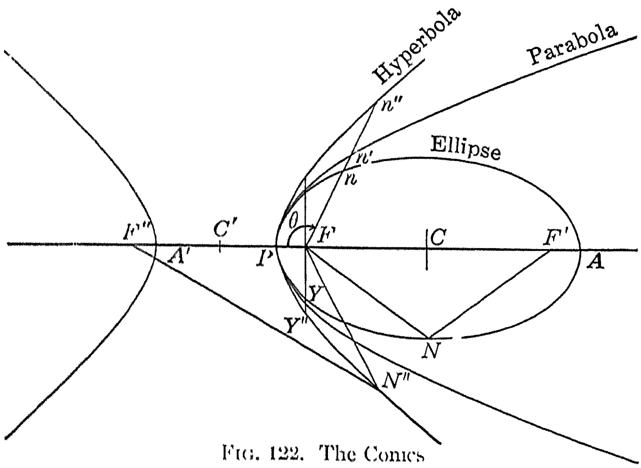


Fig. 122. The Conics

on the curve from the two foci equals the major axis, that is, $F''N'' - FN'' = PA'$. A second branch of the curve surrounds the "empty focus," F''.

The parabola, like the hyperbola, fails to return into itself; but its two portions, though still gradually separating, become more and more nearly parallel. It has but one accessible focus, and may be regarded either as an ellipse, with its second focus, F', removed to an infinite distance, and therefore having an infinite major axis, or, equally well, as a hyperbola of which the second focus F'' is pushed infinitely far in the opposite direction, so that it has an infinite (negative) major axis.

In the ellipse the *eccentricity* (FC/PC) is *less than unity*; in the hyperbola it is *greater than unity* (FC'/PC'). In the parabola it is *exactly unity*; in the circle it is *zero*.

The eccentricity of a conic determines its form. All parabolas are of the same form, differing only in size, as do circles. Ellipses

313. Equation for the Orbital Velocity. The velocity of one body relative to the other, at the distance r, is given by the equation

$$V^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right), \tag{7}$$

where a is the semi-major axis of the orbit and G the constant of gravitation. This equation is often called the *equation of energy*, since it expresses the conservation of energy.

The velocities of m and M, relative to the center of gravity, are respectively $V_1 = MV/(M+m)$ and $V_2 = mV/(M+m)$. The kinetic energy of the two bodies is $\frac{1}{2}(mV_1{}^2 + MV_2{}^2)$, which is readily found to be $\frac{1}{2}\frac{mM}{m+M}V^2$. Equation (7) may now be written

$$\frac{1}{2}\frac{mM}{m+M}V^2 - \frac{GMm}{r} = -\frac{GMm}{2a}.$$

The first term is the total kinetic energy of the orbital motion, the second the potential energy of gravitation, and the sum of these is constant.

314. The Parabolic Velocity. For a parabola a is infinite, and if the velocity in this special case is called U, equation (7) of the last section becomes

$$U^2 = 2G\left(\frac{M+m}{r}\right). \tag{8}$$

A particle projected with this velocity U at the distance r will move in a parabola, whatever be the direction of projection. This is therefore called the *parabolic velocity*.

If the velocity V exceeds the parabolic velocity U, a comes out negative and the orbit is a hyperbola; but if V is less than U, a is positive and the orbit is an ellipse. In the former case the particle never comes back, but in the latter it returns at regular intervals. The parabolic velocity is therefore often called the *velocity of escape*.

The velocity of escape at the surface of any body may be computed from (8), neglecting the mass m of the small body which is supposed to be in motion. Thus, in the case of the earth $M = 5.97 \times 10^{27}$ g., $r = 6.37 \times 10^8$ cm., and $G = 6.67 \times 10^{-8}$, whence $U = 1.13 \times 10^6$ cm./sec., or 11.3 km./sec. For the sun, for which M is 332,000 times as great and r 109 times, U is $\sqrt{332000/109}$, or 55.2 times as great, that is, 622 km./sec. For the moon it comes out 2.4 km., or not quite 8000 ft./sec. A body

This embodies Kepler's third law, and shows that all planets moving in ellipses which have the same major axis will have the same period, whatever the eccentricities of their orbits.

Strictly speaking, the values of a^3/t^2 for the different planets should not be equal, but proportional to $M + m$, where M is the mass of the sun and m that of the planet in question, but corrections due to perturbations by the other planets (§ 325) may be larger than this.

317. Expression for the Areal Velocity. By the geometry of the ellipse $b = a\sqrt{1-e^2}$, and also, if p is the semi-parameter of the ellipse (FY', Fig. 123), $p = a(1-e^2) = b^2/a$. The expression for the areal velocity may then be written

$$A = \tfrac{1}{2}\, p^{\frac{1}{2}} \sqrt{G(M+m)}\,;$$

that is, the *areal velocity is proportional to the square root of the semi-parameter.*

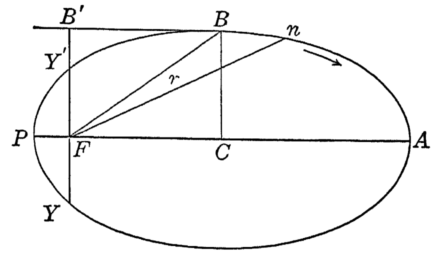


FIG. 123. An Elliptical Orbit

The sun is at the focus F; $FB = PC = a$, the semi-major axis; and $FB' = CB = b$, the semi-minor axis. The semi-parameter, p, is $FY' = FY$. The eccentricity, e, is FC/PC, which is less than unity. The radius vector r is measured from F to any point n on the orbit; and the angle PFn is θ, the true anomaly. The motion is in the direction of the arrow

All orbits for which the parameter YY' is the same will therefore be described with the same areal velocity. This is true not only of elliptic but of parabolic and hyperbolic orbits, and makes it possible to compute the motion in these, although such motion is not periodic and Kepler's third law is therefore inapplicable.

The principles summarized above have many important applications. We will mention two.

318. Calculation of the Position of a Body at Any Time: Ephemerides. If we know the orbit, the areal velocity A, and the time T at which the body passed perihelion, all that is necessary in order to find its position in its orbit at any other time t is to know how to draw a radius vector Fn (Fig. 124) such that the area between this line, the orbit, and the radius FP to the perihelion is equal to $A(t-T)$. This is known as "Kepler's problem." For a parabolic orbit it leads to a cubic equation;

360° as the time interval $t_2 - t_1$ is of the period. If this relation is satisfied, our guess at the distance is right; if not, it is wrong, and we proceed to further trials, which soon lead to the solution.

Since the actual orbit is not a circle but an ellipse, this process will give only approximate results, but it is often actually used to predict the motion of a newly discovered asteroid for a few weeks and to facilitate further observations.

The determination of an elliptic orbit requires *three* observations and the determination of *two* geocentric distances E_1P_1 and E_2P_2 (preferably for the first and last observations). If these distances are given, and the points P_1 and P_2 thus fixed, the whole orbit of the planet around the sun can be determined, and the direction of the planet from the earth at the time of the middle observation can be calculated. The two conditions that the calculated right ascension and declination shall agree with the observed values suffice to determine the two geocentric distances. Equations can be set up which, though complicated, make the solution much shorter than it would be by trial and error. In some cases (for example, when the planet is in the plane of the ecliptic) a fourth observation is required for a solution.

If the orbit is a parabola or a hyperbola, this general solution will show it and will determine the eccentricity. Comets usually move in almost parabolic orbits. Therefore, when computing the orbit of a new comet, it is assumed that it is a parabola, which considerably simplifies the calculations. If the orbit is really an ellipse, the calculated parabolic orbit cannot be made to represent both the right ascension and the declination for the middle observation, and the general method must be employed.

The calculation of an orbit from three observations takes a skilled computer two days or sometimes less. The novice may take as many weeks, most of his time being occupied in finding and correcting the numerical mistakes which are only too easy to make.

320. Radiation Pressure and its Effects. For very small bodies the effects of the sun's attraction are modified by those of another force, radiation pressure (§ 553). A beam of light falling on any body exerts a force in the direction of the incident light. This force is proportional to the intensity of the light and to the cross-section A of the beam which is intercepted by the body.

6. If Jupiter were reduced to a particle, how much would its period be lengthened? (Consider its mass to be 1/1048 of the sun's, and see § 316.)

Solution. Let x be the new period; then

$$x^2 : t^2 \frac{1049}{1048} = r^3 : r^3 = 1 : 1,$$

since r is not changed; whence

$$x = t\left(\frac{1049}{1048}\right)^{\frac{1}{2}} = t(1 + \tfrac{1}{2} \times 1/1048 + \text{etc.}) = t(1 + 1/2096) \text{ very nearly.}$$

But $\qquad t = 4332.6$ days, and $(x - t) = \dfrac{4332.6}{2096} = 2.067$ days. *Ans.*

7. How much longer would the earth's period be if it were a particle?
Ans. 1/660,000 of a year, or 47.8 sec.

8. If the sun's mass were a hundred times greater, what would be the parabolic velocity at the earth's distance from it (§ 314)?
Ans. Ten times its present value, that is, 420.9 km./sec.

9. If the sun's mass were reduced 50 per cent, what would be the parabolic velocity at the distance of the earth? *Ans.* 29.76 km./sec.

10. If the sun's mass were to be suddenly reduced by 50 per cent or more, what would be the effect upon the now practically circular orbits of the planets? (See § 315.)

11. What would be the effect upon the orbit of the earth if the sun's mass were suddenly doubled? *Ans.* It would immediately become an eccentric ellipse, with its aphelion near the point where the earth was when the change occurred.

12. How long would a comet, which is moving in a parabolic orbit with a perihelion distance of one astronomical unit, take to move through 90° from perihelion (from P to Q in Fig. 126)?

Ans. The velocity of the comet at perihelion is $\sqrt{2}$ times that of the earth in its orbit, and its areal velocity is also $\sqrt{2}$ times that of the earth. The latter is π "square astronomical units" per year (since this is the area of the earth's orbit), so that for the comet $A = \pi\sqrt{2}$. By well-known properties of the parabola $SQ = 2\,PS$, and the area of the sector SPQ is 2/3 that of the rectangle $PRQS$, or 4/3 of a square astronomical unit. The time in question is therefore $4/(3\,\pi\sqrt{2})$ years, or very nearly 3/10 of a year (109.61 days).

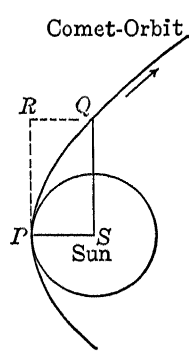


FIG. 126. Perihelion Passage of a Comet

13. Show that a comet moving in a parabola with perihelion distance q will take $109.61 \times q^{\frac{3}{2}}$ days to move through 90° from perihelion.

greatly affect its motion, and its path will be nearly, though not quite, an ellipse such as A (Fig. 127). Suppose, however, that M makes a close approach to Jupiter, as at J in the figure. The planet will then attract it strongly and may greatly change its velocity relative to the sun, so that after leaving Jupiter's vicinity it will pursue quite a different orbit B. Moving in this orbit, M will return at regular intervals to the point where it met Jupiter, but unless its new period happens to be the same as Jupiter's, at first it will not find the planet close by. Sooner or later, however, Jupiter and the meteorite will return to the point of encounter at very nearly the same time. A second close approach will take place, and the orbit will again be greatly changed.

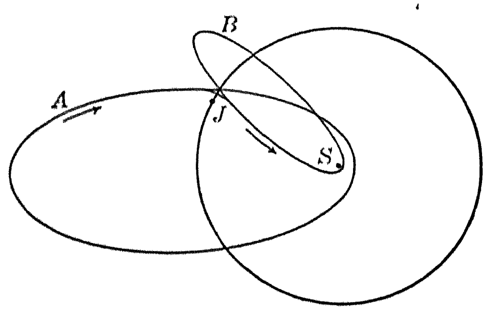


FIG. 127. Problem of Three Bodies

If the meteorite's velocity is this time sufficiently increased, it may be sent off in a parabola or hyperbola, never to return; if not, it will be diverted into still another elliptic orbit and will pursue this until another encounter with Jupiter takes place. A very small difference in the circumstances of the first encounter (perhaps a few miles of difference in the minimum distance) will lead to greater differences in the size and period of the second orbit. After the dozen revolutions or more which may elapse before the second encounter, this will result in still greater changes in the third orbit, and so on.

Any general formula that would be capable of taking into account all the possible sequences of successive encounters would have to be of almost inconceivable complexity.

Two facts — that the attraction of Jupiter would be continually modifying the meteorite's orbit, to a less degree, between the close encounters, and that the orbits are, in general, not in one plane — add further complexity to the problem. If M were another planet, of mass comparable to Jupiter's, the changes in Jupiter's orbit would also have to be taken into account, and

to improved values of the forces, from which a third approximation to the motions and positions may be derived, and so on.

In practice this process is rapidly convergent. The second approximation is very nearly sufficient for the smaller planets, and the third leaves little more to do even in the case of Jupiter and Saturn.

325. Perturbations. A planet may therefore be regarded as moving in an ellipse about the sun but having this motion modified by the attraction of the other planets. The resulting changes in its position and orbit are called *perturbations*. Though the motions of the planets are thus technically said to be "disturbed," they are actually just as natural and regular (controlled by law) as in the simpler case of undisturbed motion, only they are much more complicated.

For bodies which, like the planets and their satellites, move in nearly circular orbits it is possible to obtain analytical expressions for these perturbations, from which their values at any desired time may be computed. Such expressions are called *general perturbations*. They appear in the form of infinite series, and a great number of terms have usually to be included to obtain accurate results. The calculation of these perturbations is highly intricate.

When, as in the case of comets, the orbit is very eccentric, the perturbations must be computed by quadratures, step by step, for so long an interval as is necessary; such values are called *special perturbations*.

326. Perturbations of the Planets. The actual motions of the planets may best be described as follows:

(1) The orbits of the planets are not fixed but gradually change in eccentricity, inclination, etc., — so slowly, however, that during any one revolution the orbit suffers very little alteration.

(2) A planet, in its motion, does not exactly follow Kepler's laws, but, if compared with an ideal planet which strictly obeys these laws, is continually being shifted, forward or backward, toward or from the sun, above or below the orbit plane, though never to any great distance.

Changes of the first type are called *secular perturbations*. They depend on the relative positions of the planetary orbits. Those

Another effect of perturbations is that the relation between the period and the mean distance of a planet is not exactly that given by Kepler's third law, even when the mass of the planet itself is taken into account. On the average, planets inside the given planet's orbit increase the central attraction; those outside diminish it, increasing the period. Thus, the period of Jupiter

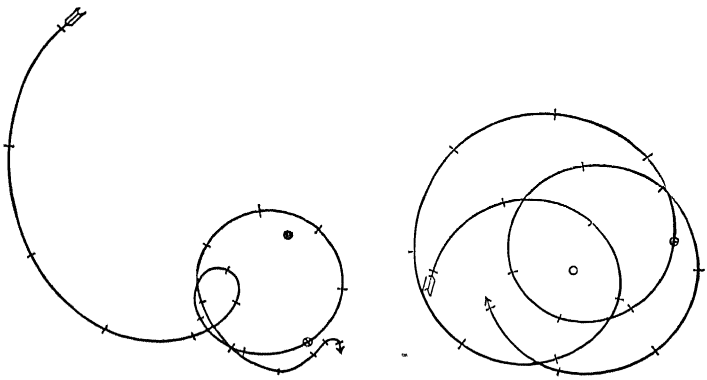


FIG. 128. Secular Perturbations of the Earth's Orbit

The calculated motions, for 200,000 years, of the center of the earth's orbit, in its own plane, are shown in *A*; and those of the pole of the ecliptic, on the celestial sphere, in *B*. The positions at intervals of 10,000 years are marked by short lines, and those at the present time by small circles. The black dot in *A* represents the sun, and the isolated circle in *B* represents the pole of the invariable plane. The earth's orbit itself, on the scale of *A*, would be about 40 inches in diameter. The eccentricity of the orbit is now 0.016 but is diminishing and will reach a minimum value of 0.003 about 24,000 years hence. The inclination to the invariable plane is now 1° 35′ and is also diminishing. It will reach a minimum value of 47′ in about 20,000 years.

is one and one-half hours longer than it would be if no other planets existed, while that of Saturn is nearly a week shorter.

329. Perturbations of Asteroids. Many asteroids have orbits of high inclination and eccentricity, which pass relatively near to Jupiter. Their perturbations are correspondingly great, and the theory of these is very complicated.

If the period is very nearly an exact sub-multiple ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, etc.) of that of Jupiter, the perturbations may gradually increase to very large values, as they reënforce each other in successive revolutions. It is significant that there are gaps in the distribution of the asteroids for just these values of the period.

The invariable plane, and not that of the earth's orbit, is the really fundamental plane of reference for the solar system It is inclined 1° 35′ 8″ to the ecliptic of 1850, and has its ascending node in longitude 186° 9′. It is intermediate between the planes of the orbits of Jupiter and Saturn, and nearer the former.

332. Definitive Orbits. In making a precise determination of the orbit of a planet or a comet all reliable observations are first collected, corrected, so far as possible, for the systematic errors of the various observers, and combined into "normal," or mean, places, giving the more accurate observations higher weights. These positions are compared with those calculated from a preliminary orbit. The outstanding deviations arise partly from perturbations, partly from errors in the elements of the preliminary orbit, and partly from errors of observation. The perturbations are computed and allowed for, and corrections to the orbital elements are then determined by the method of least squares (§ 124), so as to leave the smallest possible discrepancies to be attributed to errors of observation. The process is very laborious.

The calculations may be designed to find the *osculating orbit*, that is, the orbit which the body would pursue if, at a specified instant, all the planets were annihilated and it moved thereafter under the attraction of the sun alone. This is usually done for comets and often for asteroids. If the general perturbations have been computed, the *mean orbit* may be found, that is, the orbit which the body would follow if its periodic perturbations were abolished. This calculation is still more laborious but has been made for all the major planets and, with less precision, for many asteroids.

333. Planetary Tables. The positions of the planets given in the *Nautical Almanac* are derived from *tables* in which all known perturbations are taken into account. The design and calculation of such tables demands great mathematical knowledge and skill, but their use, with the aid of the "precepts" given in connection with them, is fairly simple.

Certain quantities called arguments, which depend on the positions of the planets in their orbits, are first calculated, and with these arguments the tables are entered which give the perturbations. By adding these to the values given by Kepler's laws (which are taken from other tables) the planet's longitude, latitude, and distance (all referred to the sun) are found. The tables of Jupiter, for example, cover ninety-one quarto pages. The planet's longitude is found by adding thirty quantities taken from them.

It is noteworthy that exact calculations can be made of the motion of a body (for example, a comet) when we have no idea what its mass is and consequently do not know the force of attraction ($f = GmM/r^2$); for the acceleration ($f/m = GM/r^2$) is accurately known.

The effects on the moon's orbit are very similar in kind to the perturbations of the planets, but greater in degree.

335. Motions of the Perigee and Node: Effect on the Length of the Month. The eccentricity of the moon's orbit, on the average, remains the same, but the *line of apsides advances*, though not continuously; so that, on the average, it completes a revolution in 8.8503 years (according to Brown). The inclination too is subject only to periodic changes, but the *nodes regress* at an average rate of a revolution in 18.5995 years.

The disturbing force, on the average, is away from the earth, so that the earth's effective attraction is diminished, and *the month is longer*, by one part in 734, or about 53 minutes, than it would be if the moon's motion were undisturbed.

These effects are clearly analogous to the secular perturbations of the planets.

336. The periodic perturbations are large and extremely numerous. According to Professor Brown's calculations (the latest and most exhaustive) there are 155 periodic terms in the expression for the moon's longitude with coefficients exceeding 0″.1, and more than 500 smaller ones, which, though individually insignificant, may at times add up to a sum which is not negligible; so that they must all be considered if we wish to compute the longitude to 0″.1. Each of these terms has its own period. The number of sensible terms in the latitude is about half as great; and almost 150 have to be taken into account in computing the parallax.

337. The Principal Terms. The nature of the results may best be understood by examining the first few terms in the expressions for the moon's longitude and parallax. These may be written

$$\lambda = L + 377' \sin l + 13' \sin 2l + 76' \sin(2D - l) + 40' \sin 2D - 11' \sin l' + \cdots,$$
$$\pi = 3424'' + 187'' \cos l + 10'' \cos 2l + 34'' \cos(2D - l) + 28'' \cos 2D + \cdots.$$

Here L is the moon's mean longitude, or the angular distance of an imaginary uniformly moving mean moon from the vernal equinox, l, the distance of the mean moon from the mean perigee; D, its distance from the mean sun; and l', the distance of the latter from its perigee (which corresponds to the earth's perihelion).

diminish. After about 24,000 years (Fig. 128) the effect will diminish to zero and then become reversed.

As a result of this the moon slowly forges ahead of the position computed with a uniform period, to an extent that increases as the *square* of the time, since in a longer interval there are more months and the average length of each is less. The theoretical change amounts to only 6″.01 at the end of a century (according to Brown), but in 2000 years this becomes 40′, a quantity great enough to be detected even from the rough records of ancient observations.

More than two hundred years ago Halley found this secular acceleration of the moon's motion by comparing ancient with modern eclipses. Fotheringham (1915), from a thorough discussion of all the records of ancient observations, finds an acceleration of 10″ 3 per century. The difference between this and the theoretical value is undoubtedly real, and must arise from a gradual slowing of the earth's rotation and lengthening of the day (§ 355). This explanation is confirmed by the fact that the ancient eclipses indicate an apparent secular acceleration of the sun at the rate of 1″.5 per century, which can be explained only by an increase in the length of the day.

339. The Lunar Theory. The precise calculation of the motion of the moon is one of the most difficult and laborious tasks in the whole realm of applied mathematics. With respect to the sun's action, the agreement of the results of Hansen, Delaunay, and Brown (who worked by altogether different methods, each spending between ten and twenty years on the problem) is conclusive evidence that the calculations are correct and complete.

The even more complicated planetary action has now been fully computed by investigations whose complexity Professor Brown compares to "playing chess in three dimensions blindfolded," and in which several thousand terms were examined to see whether they were of sensible magnitude. Brown's new *Tables of the Moon* fill three quarto volumes, totaling more than 360 pages.

340. Outstanding Irregularities in the Moon's Motion. After the influence of the attraction of all known bodies has been allowed for, there remain unexplained and very remarkable discordances between the computed and observed longitudes of the

The oblateness ϵ depends also on the planet's internal constitution. For a homogeneous planet it may be proved that $\epsilon = \frac{5}{4}\ \phi$. For a planet in which almost the whole mass is concentrated into a small lump at the center, $\epsilon = \frac{1}{2}\ \phi$. For several of the planets ϵ may be found by direct measurement, and ϕ may be computed by the equation given above. The ratio ϵ/ϕ is found to be 1.14 for Mars, 0.97 for the earth, 0.76 for Jupiter, and 0 62 for Saturn. It is evident, therefore, that Mars must be of nearly uniform internal density, while Jupiter, and especially Saturn, must be very much denser at the center than near the surface.

The force of gravity at the surface depends on both the ellipticity and the internal constitution. If W is the fraction by which gravity at the pole exceeds that at the equator, then

$$W = \tfrac{5}{2}\ \phi - \epsilon, \tag{10}$$

as was proved by Clairaut. This equation is used in finding the earth's ellipticity from measures of gravity (§ 138).

342. Perturbations of Satellites. For a close satellite of an oblate planet the principal disturbing force arises from the attraction of the equatorial protuberance, or bulge. If no other disturbing forces are at work, the satellite revolves with a fixed mean distance and period, and the eccentricity of the orbit, and its inclination to the plane of the planet's equator, remain practically constant. The line of apsides advances uniformly, and the nodes on the equatorial plane regress at the same rate, completing a revolution in the same period.

The period of this revolution is

$$\frac{a^2}{r^2} \times \frac{T}{\epsilon - \frac{1}{2}\ \phi},$$

where a is the satellite's mean distance and T its period, while r, ϵ, and ϕ have the meanings defined above. The ellipticity of a planet may thus be found from observations of its satellites.

For those satellites upon which the action of the sun is sensible the nodes regress upon a "proper plane" which lies between the planes of the planet's equator and orbit and is a sort of compromise between them The inclination of the satellite's orbit to this plane varies but little. For remote satellites, like the moon, this proper plane almost coincides with that of the planet's orbit, and the principal perturbations are due to the sun.

Further evidence that the moon is largely responsible for the tides is found in the fact that when the moon is in perigee (nearest the earth) their range is nearly 20 per cent greater than when it is in apogee. The greatest range of all happens when the moon is new or full at the time when it is in perigee.

The "establishment of a port" is the mean interval between the time of high water at that port and the next preceding passage of the moon across the meridian. The establishment of New York, for instance, is $8^h\ 13^m$, — on the average, high water occurs there $8^h\ 13^m$ after the moon has crossed the meridian; but the actual interval varies fully half an hour on each side of this.

In North Atlantic waters the morning and afternoon high tides are about equally high, and the low tides equally low, but in many seas, as in the Gulf of Mexico and the North Pacific, there is a marked *diurnal inequality* in the heights, coupled with considerable irregularity in the times of high and low water. Indeed, in extreme cases there is but one high water and one low water a day. This inequality occurs only when the moon is far north or south of the equator. When the declination is zero, there are two equal tides daily.

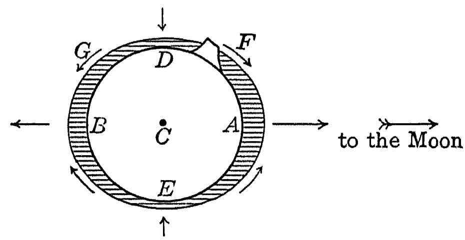


FIG 130 The Tide-Raising Force

All these complicated phenomena are readily explicable by the effects of the disturbing forces, due to the sun and moon, on the water of the oceans.

345. The tide-raising force is very similar in nature to the disturbing force of the sun upon the moon, which has already been discussed (§ 334). The moon, for example, attracts the ocean at A (Fig. 130), directly beneath it and nearer to it, more powerfully than it attracts the solid mass of the earth, and so tends to pull the two apart. The ocean at B, on the far side, is less strongly attracted than the mass of the earth, and so the disturbing force again tends to separate the two. At D and E the moon's attrac-

The sun's tidal force would raise a similar but lower tide-wave with its crests under the sun and opposite to it, which would be superposed on the lunar wave and modify its effects.

347. Explanation of the Principal Tidal Phenomena. At new and full moon the lunar and solar high tides would come together, and the range of level would be unusually great. At the quarters the solar high tide would coincide with the lunar low tide, and would partly fill up the depression, producing a decreased range. This explains the spring and neap tides. The greatest and least ranges should be in the ratio of $(11 + 5)$ to $(11 - 5)$, or as 8 to 3. The observed ratio is usually not quite so great.

When the moon is in perigee, it is fully 10 per cent nearer than at apogee, and its tidal force is about 30 per cent greater, which immediately explains the increased range of the tides at perigee.

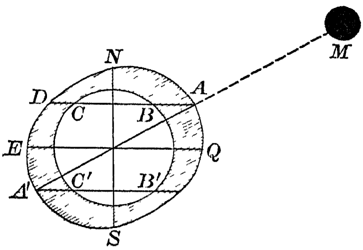


FIG. 131. The Diurnal Inequality

When the moon is farthest north of the celestial equator (*EQ*, prolonged, Fig. 131), the tidal summits will be at *A* and *A'*; the tide which occurs at *B* when the moon is overhead will be great, while the tide in the corresponding southern latitude at *B'* will be small. Twelve hours later, when the earth's rotation has carried the observers to *C* and *C'*, the tide at the northern station will be small and the one at the southern station will be large. The existence of a diurnal inequality in the tides is thus explained. For points much nearer the pole than *B* there will be but one high tide and one low tide a day, while on the equator, at *E* or *Q*, the two tides should be equal.

When the moon is on the celestial equator, *A* and *A'* are on the earth's equator, and everywhere there will be two equally high tides a day, once more accounting for an observed fact.

348. Complexity of the Actual Problem. This simple theory (worked out by Newton) accounts for the general behavior of the tides very well, but it breaks down in details It does not explain why high tide, at different points, comes at all sorts of different intervals after the moon's transit, or why the range of the tides is

hours. After such a force has acted long enough for a "steady state" to be established, the surface of the ocean will be thrown into oscillations. The character of these oscillations — whether they are simple motions, like those in a small lake, or complicated ones, with several regions in which the water rises at the same time, separated by others in which it falls — will depend on the size, shape, and depth of the sea and on the range of level. The time of high water will differ from place to place, but the period will be exactly that of the impressed force.

Similarly, the lunar tide-raising force, if acting alone, would set up a series of oscillations at intervals of $12^h\,25^m.2$ with a greater range and a somewhat different "pattern" of regions of high and low water. These two oscillations are called the solar and lunar semi-diurnal tides. Under the combined action of the moon and sun they are superposed without sensibly modifying one another, and their combination produces the spring and neap tides.

The other variations in the tide-raising force may be taken into account by introducing additional periodic terms. For example, the main part of the changes due to the moon's varying distance may be allowed for by superposing on the lunar semi-diurnal tide, the period of which is $12^h\,25^m\,14^s$, another tide of about 1/5 the range, and of period $12^h\,39^m\,30^s$. This falls into step with the first whenever the moon is in perigee, and increases the range, while at apogee it is out of step and diminishes it. The lunar diurnal inequalities may be represented by a pair of oscillations, of period nearly one day, which annul each other when the moon is on the equator, but reënforce each other when it is farthest north or south.

351. Analysis and Prediction of the Tides. When a record of the actual fluctuations of the water-level at a given port for a year or more has been obtained (which may readily be done by a self-registering tide-gauge), the ranges and phases of the separate periodic changes of level are not hard to find. For example, taking readings made at noon on every day of the year, the lunar tide will be sometimes high and sometimes low, and its effects will practically disappear in the average, while the solar tide will always be the same. Taking averages of the readings at 1 P.M., 2 P.M., etc., the course of the solar tides can be found. Tides of other periods can be treated similarly.

354. Direct Measurement of the Tide-Raising Force; Rigidity of the Earth. The tidal force, even when the moon and sun act together, is so small that it is very difficult to detect by direct observation. The problem was solved by Michelson in 1913, by remarkably simple means. A carefully leveled line of iron pipe 500 feet long was buried in a trench; the pipe was half filled with water and sealed air-tight. At each end was a window, through which the water-level could be observed by a microscope. The attraction of the sun and moon produced tides within this pipe, the water rising at one end and falling at the other. The height of these tides amounted at most to less than 1/1000 of an inch, but this could be measured to about 1 per cent. All the phenomena of the tides — springs and neaps, perigee tides, and diurnal inequality — were beautifully exhibited. Two pipe lines, one running north and south and the other east and west, permitted a study of the tidal force in both directions.

Discussion of such measures showed that the times of high water, for the various component periodic oscillations, and their relative heights, agreed admirably with theory; but the heights were in all cases only 69 per cent of the predicted heights.

The reason for this is that the solid earth is elastic and is deformed by the tidal forces, so that its surface rises, falls, and tilts as that of a liquid sphere would do, but to only 30 per cent as great an extent. From this it can be shown that the rigidity of the earth, as a whole, must be about equal to that of steel. It appears, also, that the earth is almost perfectly elastic; there is no evidence of any lag in the action, as there would be if there were any slow permanent yielding, such as would occur in a semi-solid or viscous mass. The actual rise and fall of the earth's surface amounts to about nine inches at the time of spring tides.

355. Tidal Friction: Effects on the Earth's Rotation. In the deep sea the tidal motions of the water are slow and there is little friction, but near the land great masses of water flow in upon the shallows and out again to sea, with a large amount of fluid friction, and this involves the expenditure of a considerable amount of energy, which is converted into heat.

This energy must be derived mainly from the earth's energy of rotation, and the necessary effect is to lessen the rate of rotation

coast, but such small units are now unprofitable. There are few places where the height of the tides and the volume of the flow would justify a modern hydro-electric development, and the fact that high water comes at a different time each day complicates the economic side of the problem, so that no such plants are in operation, though some are projected

358. Effects of Tidal Friction on the Moon's Orbit. Tidal friction has also important effects on the motion of the moon, less obvious than those on the earth's rotation. As the earth's rotation grows slower the angular momentum associated with it diminishes. But angular momentum is indestructible (§ 302); it may be transferred from one body to another in a system, but cannot disappear. If the earth and the moon were wholly isolated, all the angular momentum lost by the earth would be gained by the moon and would alter its orbital motion. Now the angular momentum for two bodies moving in a Keplerian ellipse is proportional to the square root of their distance at a point 90° from the perigee, that is, of the semi-parameter p of the orbit (§ 317). Hence, as the moon gains angular momentum its orbit must expand, it must recede from the earth, and the month must grow longer.

At the present time the angular momentum of the orbital motions of the earth and moon about their center of gravity is 4.82 times that of the earth's rotation. Taking the latter as a unit, the whole momentum is 5 82 If the length of the day should change from its present value d_0 to d, the rotational momentum would change from 1 to d_0/d, since it is proportional to the rate of rotation; and if the moon's distance, measured as defined above, should change from p_0 to p, the orbital momentum would become $4.82\sqrt{p/p_0}$. Hence, if it were not for the solar tides, we should have

$$\frac{d_0}{d} + 4.82\sqrt{\frac{p}{p_0}} = 5.82$$

throughout the whole course of the changes, and this equation would give the moon's distance corresponding to any given length of the day.

The solar tides also slow the earth's rotation, and transfer angular momentum to the earth's orbital motion around the

transferred to the orbital motion of the system around the sun, until at last the moon comes back close to the earth. Then (according to Jeffreys) it may be torn to pieces by the tidal forces (§ 461) and form a ring around the earth like those of Saturn. The sun itself, however, may have ceased to shine before time enough has passed for these exceedingly slow changes to be completed.

RELATIVITY

Within the last two decades many fundamental physical conceptions have been modified by the advent of the *theory of relativity*, first propounded by Albert Einstein (1905). A general discussion of this theory would lead far outside the range of this book, but certain of its applications, and some of the most important observational tests of the theory, fall within the field of astronomy and must be mentioned here.

360. The principle of relativity is the postulate that the laws of nature are such that all physical phenomena depend only on the *relative* positions and motions of the bodies concerned (including everything on which observations are made, even if it be a distant star), and are quite unaffected by any uniform rectilinear motion which may be common to them all.

With regard to the motions of material bodies this follows immediately from Newton's laws of motion and is confirmed by everyday experience. Thus, the earth's orbital motion (which is nearly enough rectilinear to serve as an illustration) has no influence on the relative positions or motions of objects on its surface, nor does the rapid motion of the solar system through interstellar space (§ 740) have the least influence on the paths which the planets pursue relatively to the sun. (The displacement of Aristotle's views by Newton's was therefore the first and greatest triumph of relativity.)

The modern question was, whether experiments on *light* (or electrical experiments of certain types) could detect uniform rectilinear motion, — if the observer and all his apparatus, for example, were inclosed in a great box, moving with him, which shut out all influences from the universe outside. Though some experimental difficulties and disagreements remain, the general outcome of the investigations of the past generation is in favor of the conclusion that here too the principle of rela-

and all — is falling freely in a uniform gravitational field, in which the gravitational force is equal and parallel at all points. This box, and everything in it, will be accelerated at the same rate, so that the relative motions of material bodies within it, even according to ordinary mechanics, will not be affected in the least by the external force.[1] Once more the question has to do with optical (and electrical) phenomena; relativity insists that these also are unaffected; and the full development here of the consequences of this principle leads to strange conclusions.

It appears that, in the vicinity of material bodies which exert a gravitational influence, the geometry of space may be represented as not of the simple type studied by Euclid The result is that in the vicinity of a large mass the natural path of a freely moving body is not straight but curved, — coinciding, in fact, with the orbit which is ordinarily spoken of as being due to the action of the gravitational force due to the large mass. Thus the motion of a planet in its elliptical orbit, and that of a body in a straight line in empty space, remote from even the stars, are, from the new standpoint, regarded as equally natural, neither requiring any specific explanation or the action of any force. When motion of either type is prevented, as when a stone is hung up by a string or whirled rapidly around in a circle at the end of it, the "gravitational force" which is felt in the first case, and the "centrifugal force" in the second, both arise from the fact that the stone is not permitted to follow its natural path.

In all but a very few instances the consequences of the new theory agree with those of Newton's laws, far within the accuracy of the most refined observations, so that the older and far simpler mathematical methods are still to be employed in practice. But there are three cases in which the results differ measurably, and in each of these the observations are decisively in favor of the new theory.

362. The Motion of the Perihelion of Mercury. According to Einstein the orbit of a planet about the sun (if not disturbed by the attraction of other planets) will be sensibly elliptical in form, with the sun at one focus; the line of apsides, however, will not

[1] It is only because the attractions of the sun and moon on the earth are *not* equal and parallel at all points that they raise the tides.

The only way to observe this deflection is to photograph the stars surrounding the sun during a total solar eclipse. The bending of the light rays will make the stars appear farther from the sun's center than they otherwise would (Fig. 132). By comparing the photographs taken during the eclipse with others taken under like conditions a few months earlier or later, when the sun is out of the way, the gravitational deflection can be measured.

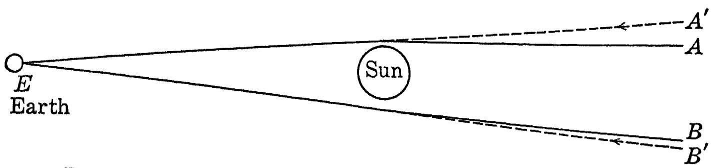


FIG. 132 Deflection of Light Rays in Sun's Gravitational Field

Rays from A and B are deflected inward, so that to an observer on the earth at E the rays appear to come from A' and B', farther from the sun The ray which passes nearer the sun is deflected more The effect is enormously exaggerated in the diagram

The observations are extremely delicate. Great care must be used that the camera is properly adjusted, and that the adjustments are the same when the comparison plates are taken. The stars must be at the same altitude in the sky in the two cases, so that the effects of refraction may be the same. When these precautions are employed, and when the number of stars shown on the plates is considerable (thus permitting a determination of the small outstanding differences of adjustment from the plates themselves), reliable results can be obtained.

The test was first made by two English parties at the eclipse of 1919. Observations in Africa were much disturbed by clouds, but those in Brazil were successful in spite of some instrumental troubles. The plates gave conclusive evidence of the existence of the deflection, and determined the amount at the sun's limb (from the observed values farther out) as $1''.98$, which agreed with the predicted value within the error of observation.

Still more definite results were obtained at the Australian eclipse of 1922, especially by the Lick expedition. Every precaution was taken in the construction of the instruments, the exposure of the plates, and their measurement and reduction. More than ninety stars were measured. The mean result from four pairs of plates gave the value $1''.78 \pm 0''.11$ for the deflection at the sun's limb (in almost exact agreement with prediction), and

CHAPTER XI

THE TERRESTRIAL AND MINOR PLANETS

MERCURY, VENUS, EARTH, AND MARS · THE ASTEROIDS · ZODIACAL LIGHT

MERCURY

365. Mercury has been known from remote antiquity, and there are recorded observations running back to 264 B.C. At first astronomers failed to recognize it as the same body on the eastern and western side of the sun, and among the Greeks it had for a time two names, — Apollo when a morning star and Mercury when an evening star. It is so near the sun that it is comparatively seldom seen with the naked eye (Copernicus is said never to have seen it), but when near its greatest elongation it is easily visible as a brilliant star low down in the twilight, varying, according to circumstances, between the stellar magnitudes (§ 688) -1.2 and $+1.1$, that is, from almost the brightness of Sirius to that of Aldebaran It is best seen in the evening at such eastern elongations as occur in March and April. As a morning star it is best seen at western elongations in September and October.

It is an exceptional planet in various ways. It is the *nearest* to the sun, *receives the most light and heat*, is the *swiftest in its movement*, and (excepting some of the asteroids) *has the most eccentric orbit*, with *greatest inclination to the ecliptic.* It is also the *smallest in diameter* (again excepting the asteroids) and has the *least mass.*

366. Its Orbit. Its mean distance from the sun is 35,950,000 miles, but the eccentricity of its orbit is so great (0.206) that the sun is 7,400,000 miles out of the center, and the distance of the planet from the sun ranges all the way from 28,550,000 to 43,350,000, while the velocity in its orbit varies from 36 miles a second at perihelion to only 24 at aphelion. Its distance from the earth ranges from about 50,000,000 miles at the most favorable inferior conjunction to about 136,000,000 at the remotest superior conjunction.

The latest calculated value of the mass of Mercury (de Sitter) is 1/8,000,000 of the sun's mass, or 1/24 that of the earth, with a probable error of about 25 per cent. With this mass, and a diameter of 5000 kilometers, the surface gravity comes out 0.27 that of the earth, and the density 0.70 that of the earth, or 3.8 times that of water, — intermediate between the densities of Mars and the moon, as seems reasonable. These results, however, are subject to very considerable uncertainty.

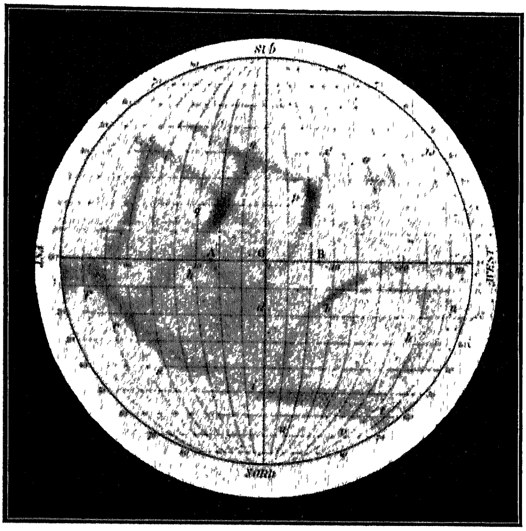


FIG. 134. Surface Detail of Mercury
After Schiaparelli

368. Telescopic Appearance and Rotation. Like the moon and Mars (which also possess solid surfaces), but unlike Jupiter, the illuminated edge of Mercury's disk is brighter than the center. Generally, of course, the planet is so near the sun that it can be observed only by day; but when proper precautions are taken to screen the object-glass from direct sunlight, such daylight observation is not especially difficult. It is only under very favorable atmospheric conditions, however, that any details can be seen on the surface of Mercury. In 1889 Schiaparelli, the Italian astronomer, announced the discovery of certain dark permanent markings there (Fig. 134). As in the case of other difficultly visible planetary details, there are considerable differences between the descriptions of the markings by different observers. Barnard described them as "very much resembling those seen on the moon with the naked eye," while Lowell drew them as almost linear.

without a telescope (Fig. 135). Since the earth passes the planet's node on May 7 and November 9, transits can occur only near those dates.

At the May transits the planet is near its aphelion and much nearer the earth than it is ordinarily, and the transit limit (corresponding to an ecliptic limit) is 2° 40′ on either side of the node. For the November transit, when the planet is nearer the sun, the corresponding limit is 4° 45′, so that transits at this node are about twice as numerous as at the other.

For the November transits the interval between two successive passages of Mercury across the sun is sometimes only 7 years, but is usually 13 years. For the May transits the 7-year interval is impossible. Twenty-two synodic periods of Mercury are nearly equal to 7 years; 41, much more nearly equal to 13 years; and 145, almost exactly equal to 46 years. Hence, 46 years after a given transit another one at the same node is almost certain.

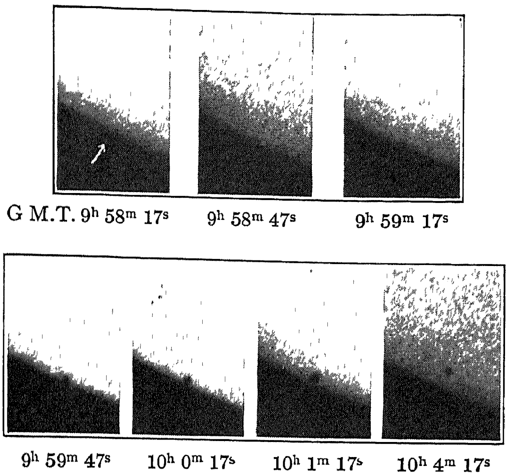


FIG. 135. Transit of Mercury, November 7, 1914

The photograph at $9^h\,58^m\,17^s$ shows a slight depression in the limb of the sun, the planet starting to enter on the disk. Second contact occurred shortly after $9^h\,59^m\,47^s$. (From photograph by Royal Observatory, Greenwich)

The first and second contacts of the transit of May 7, 1924, were visible in the United States and were well observed. The sun set with the planet well on the disk. The next transit wholly visible in the United States will occur on November 14, 1953.

A transit of Mercury furnishes an opportunity for determining corrections to the planet's place and its orbital elements.

The difference is clearly due to irradiation. The real diameter is probably not far from 7700 miles, with a probable error not exceeding 1 per cent.

According to this its *surface* is 0.95 of that of the earth; its volume, 0.92.

By means of the perturbations which the planet produces on the earth and Mars the *mass* of Venus is found to be 1/410,000 of the sun's mass, or 0.81 of the earth's. This value is uncertain by a considerable fraction of 1 per cent. The density comes out 88 per cent, and the superficial gravity 85 per cent, of the earth's. A man who weighs 160 pounds on the earth would weigh 136 pounds on Venus.

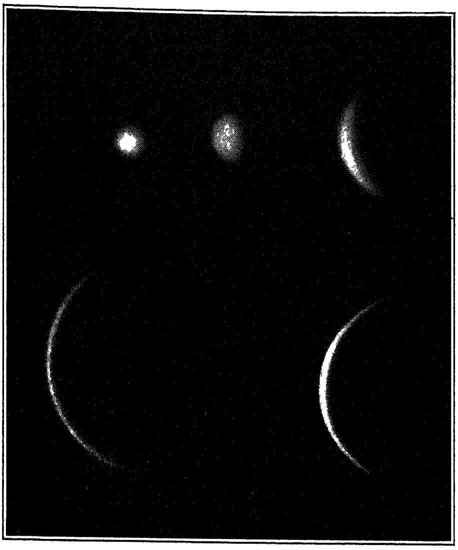


FIG 136 Venus

Photographs showing the various phases and true relative sizes of the planet's disk presented during a synodic period (From photographs by E. C. Slipher, Lowell Observatory)

373. Brightness, Albedo, and Phases of Venus. Whenever visible at all this planet appears brighter than any other, ranging from -3.3 to -4.3 in stellar magnitude (§ 688). As it moves from superior toward inferior conjunction the increase in apparent diameter at first more than makes up for the diminution of the illuminated portion of the disk, and the planet grows brighter. As the crescent begins to narrow, however, the second influence outweighs the first, and the brightness diminishes again. Maximum brightness is reached about 36 days before and after inferior conjunction, at an elongation of 39° from the sun, when the phase is like that of the moon when it is about five days old (Fig. 136). At this time the planet appears about $2\frac{1}{2}$ times as bright as when near superior

Under favorable atmospheric conditions, however, faint and ill-defined markings are often seen. Barnard described them as "large dusky spots," too elusive to reproduce in a drawing, and said that observations on different days gave the impression that they were not permanent.

375. Rotation of the Planet. The rotation period of Venus is not yet determined, mainly because of the difficulty of finding definite and recognizable markings on its surface. It was for a long time supposed, on insufficient evidence, to be a little less than 24 hours; Schiaparelli concluded that the rotation must be very slow, and that it was probable that Venus, like Mercury, keeps always the same face toward the sun.

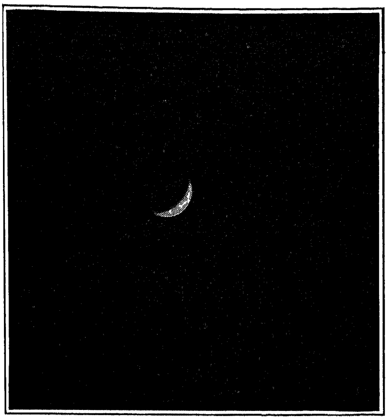


FIG. 137. Venus in the Crescent Phase

Enlarged from a photograph with the 40-inch refractor (From photograph by E E Barnard, Yerkes Observatory, June 5, 1908)

A more hopeful means of attacking the problem is found in spectroscopic determinations of the radial velocity (§ 579) of points on opposite sides of the disk. Observations at the Lowell and Mt. Wilson observatories agree in showing that the velocity of rotation is too small to measure. A rotation period less than 20 days would very probably have been detected, but one of more than 5 or 6 weeks would have escaped observation.

If the rotation period were comparable with a day, the planet should be perceptibly flattened at the poles, and the difference between the equatorial and polar diameters, when Venus is nearest us, should be about $0''.2$; but the numerous and precise measures made during transits show no evidence of oblateness.

There seems no doubt, then, that the period of rotation is longer than that of any planet except Mercury, but it is not likely that Venus keeps the same face toward the sun. Pettit and Nicholson find that considerable heat is radiated from the dark

atmosphere. The deviation of this light is, however, but a minute or two of arc, and a rare atmosphere would suffice to produce it.

It may therefore be concluded that the atmosphere of Venus above the visible surface is much less extensive and dense than the earth's, and perhaps comparable with the amount of the earth's atmosphere which lies above the high clouds. (If Venus's apparent surface is a cloud-layer, there may be any amount more below it, and for this very reason inaccessible to our investigation.)

If this atmosphere contained oxygen or water vapor in any considerable quantity, their presence could be detected by certain lines which they would produce in the spectrum of the light reflected by the planet (§ 619). St. John, after very careful observations, finds no evidence of such lines, and concludes that the amount of oxygen in the atmosphere above the visible surface must be less than 1/1000 of the quantity in the earth's atmosphere. The test for water vapor is not so delicate, and a quantity such as prevails above the highest clouds on the earth might escape detection; but 1 per cent as much oxygen as exists above these clouds could have been recognized.

377. Physical Condition and Habitability. The force of gravity is sufficient to retain an atmosphere similar to ours, and it is quite possible that there may be water vapor below the level of the clouds, through which the spectroscope is unable to penetrate. Conclusions regarding the physical conditions which prevail on the surface depend essentially on the assumed period of rotation.

If, as now appears decidedly probable, this period is some weeks in length, and if there are oceans on the planet, as seems likely from its similarity to the earth, then, as Clayden pointed out years ago, there should be ascending air currents and clouds over the sunlit side, with descending currents and possibly clearer sky on the night side. This would account for the brilliant whiteness of the surface, while the vague and fugitive markings which are seen would be naturally interpreted as thinner places in the cloud-layer. If this is the case, there may be much more atmosphere underneath the clouds, and the surface may be much warmer than the upper clouds, as is the case on the earth.

The earliest observed transit, in 1639, was seen by two persons only (Horrocks and Crabtree, in England), but the four which have since occurred, in June, 1761 and 1769, and in December, 1874 and 1882, were extensively observed by scientific expeditions sent out by the different governments to all parts of the world where they were visible. The transits of 1769 and 1882 were visible in the United States

It is not likely, however, that so much trouble and expense will hereafter be expended upon observations of transits. Other methods of determining the solar parallax have been found to be more trustworthy.

380. Recurrence and Dates of Transits. Five synodic revolutions of Venus are very nearly equal to 8 years, the difference being little more than one day; and 152 synodic revolutions are still more nearly (in fact, almost exactly) equal to 243 years. If, then, we have a transit at any time, another *may* occur at the same node 8 years earlier or later. This will be impossible 16 years before or after, and no other transit can then occur *at the same node* until after the lapse of 235 or 243 years, though a transit or pair of transits may, and usually will, occur *at the other node* in about half that time; thus, the next pair of transits of Venus will occur on June 8, 2004, and June 6, 2012.

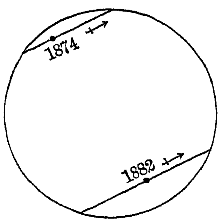


FIG. 138. Tracks of Transits of Venus

If the planet crosses the sun almost centrally, the transit will be solitary, that is, not accompanied by another one 8 years before or after. If, however, the track is more than 12′ from the sun's center, it will be accompanied by another at an interval of 8 years. Transits come thus in pairs at present, and have been doing so for several centuries; after a time this will cease to be true, and they will become solitary for another long period.

THE EARTH

Certain characteristics of the earth as a planet may best be discussed here.

381. Albedo. The only direct way of determining the earth's albedo is by measuring the brightness of the earth-shine on the moon (§ 185) compared with that of the sunlit portion. The observations are difficult, but the rather imperfect ones that have

the seasons, flanked on either side by nearly cloudless and therefore much darker belts over the desert zones near the tropics of Cancer and Capricorn, beyond which brighter, partly cloudy regions would extend to the poles.

These belts would exhibit a wealth of ever-changing details, among which the great cyclonic storms of temperate latitudes would be conspicuous as white (cloudy) areas followed by dark (clear) ones, moving eastward and lasting for a number of days.

In consequence of this eastward motion the observed average rotation period of the cloudy markings in this zone would be considerably less than twenty-four hours, while nearer the equator, in the trade-wind region, where the winds blow regularly from the northeast or southeast, the observed average rotation period would be more than twenty-four hours.

Except in desert belts there would rarely be an area as much as a thousand miles square entirely free from clouds, and the study of the details of the true surface would be greatly hampered. Even in cloudless weather the surface would be seen through a blue atmospheric veil, like that which intervenes between an observer on the earth's surface and a mountain ten miles or so away, and details could be made out only when the air was free from dust or haze. The most conspicuous surface features would probably be (1) the reflection of the sun from the ocean (under favorable conditions, by far the most brilliant thing on the planet); (2) snow-covered areas (much confused, however, by the overlying clouds); (3) deserts (if practically devoid of vegetation, yellowish or reddish in tone). The darkest parts of the surface would be the oceans (where not directly reflecting sunlight) and the great forest regions, both of which would appear of a dull blue, since most of the reflected light would come from the overlying atmosphere. Cultivated regions and grasslands would appear of a lighter tone (greener), but only the most generalized features of their distribution could be discovered.

384. In the frontispiece, which is from a painting by Howard Russell Butler, the earth is represented as seen from the floor of a crater near the moon's north pole, about three days before new moon. The most conspicuous objects on the surface are the reflection of the sun on the Atlantic Ocean off the coast of Africa,

as smoky patches by day, and every active terrestrial volcano would look somewhat the same.

Though these evidences of human endeavor would be visible from the moon, it is very doubtful if they could be distinguished from the results of what we usually call natural processes, or would present any recognizable evidence of intelligent design or purpose; and it is practically certain that no definite evidence of the existence of mankind could be detected at all by observations from Venus with instruments such as ours.

MARS

386. This planet, like Mercury and Venus, is prehistoric as to its discovery. It is so conspicuous in color and brightness, and in the extent and apparent capriciousness of its movement among the stars, that it could not have escaped the notice of very early observers.

Its *mean distance* from the sun is a little more than one and a half times that of the earth (141,500,000 miles), and the *eccentricity* of its orbit is so considerable (0.093) that its radius vector varies more than 26,000,000 miles.

At opposition the planet's *average* distance from the earth is 48,600,000 miles. When opposition occurs near the planet's perihelion this distance may be reduced to 34,600,000 miles, while near aphelion it may be as great as 62,900,000 miles. At conjunction the average distance from the earth is 234,400,000 miles.

At an average conjunction Mars is of stellar magnitude + 1.6, — about half as bright again as the polestar; at an unfavorable opposition it is twelve times as bright and of magnitude − 1.1, — not as bright as Sirius; while at the most favorable opposition it is of magnitude − 2.8, — fifty-five times as bright as at the average conjunction and brighter than any other planet except Venus.

These favorable oppositions occur always in the latter part of August (when the earth passes the perihelion of the planet) and at intervals of 15 or 17 years. The last was in 1924.

The *inclination* of the orbit is small, — about 1° 51′.

The planet's *sidereal period* is 687 days, or one year and ten and one-half months; its *synodic period* is 780 days, or nearly

391. Rotation. The spots on the planet's disk enable us to determine its period of rotation with great precision. Its *sidereal day* is $24^h\,37^m\,22^s.58$, according to the last determination by Lowell. Observations made a few days or weeks apart give a sufficiently approximate value of the time of rotation to enable one to determine, without fear of error, the whole number of rotations between two observations separated by a much longer interval. Thus a very precise determination can be effected by comparing drawings of the planet made by Huygens and Hooke more than two hundred years ago with others made more recently. The figure given above for the rotation period is not uncertain by more than a few hundredths of a second.

The rotation of the planet gives rise to a regular variation in its brightness, amounting, according to the observations of Guthnick, 1914, to about 15 per cent of the whole. The variation arises from the varying presentation of the lighter and darker spots on its surface.

392. Presentation of Regions of the Surface. The rotation of Mars is so little slower than that of the earth that an observer who notices a given marking near the center of the planet's disk (say at midnight) will see it on the following night at the same hour in almost the same apparent position. This region of the planet's surface will therefore be observable for a number of nights in succession. On account of the greater length of the Martian day, however, the region observable at a fixed hour of the night slowly works backward around the planet, by about 9° per day, and the region first observed gradually passes out of sight, and returns to a new presentation after about 40 days. Midway in this interval the opposite side of the planet is observable, while the side of the planet first considered is visible to observers on the opposite side of the earth.

393. The Inclination of the Planet's Equator to the Plane of its Orbit. This is 25°10′, according to H. Struve's careful study of the satellite orbits. Lowell, from long series of observations of the polar caps, obtains 23° 30′. The discordance between the two determinations shows that systematic errors (§ 122) must affect one or both.

The planet's disk is, as a whole, *ruddy or orange-colored* and is usually brighter around the limb, though not at the terminator, if there is any considerable phase. About three eighths of the

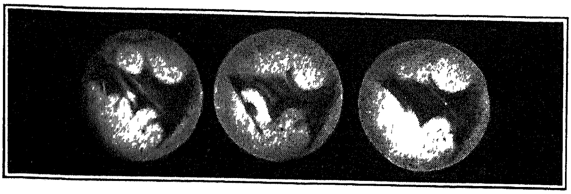
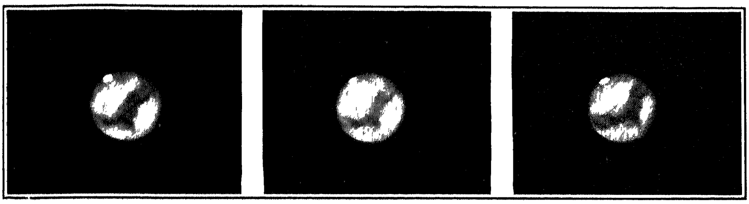


Fig 139. Mars
From drawings by Barnard

surface, however, is covered by *darker regions* of a bluish-gray or greenish shade, and at the poles appear brilliant white patches, the *polar caps*, which are usually the most conspicuous features of all.

Fig. 140, from a photograph by Professor Barnard, shows these features admirably, and also illustrates the gradual passage of the markings across the disk as the planet rotates on its axis.

South



$16^h\,46^m$ G.M.T. $16^h\,24^m$ $15^h\,24^m$

September 28, 1909

Fig 140. Mars

The rotation of the planet is evident from the comparison of the three photographs. The wedge-shaped extension (toward the north) of the large dark area is one of the most easily recognized features. It is known as *Syrtis Major*. The south pole is tilted toward the earth. It is summer in the southern hemisphere, and the polar cap has receded to a diameter of about 450 miles. (From photographs by E E Barnard, Yerkes Observatory)

The *reddish areas* show little or no change with the seasons, and are generally believed to represent the bare and, in itself, almost featureless surface of the planet, upon which the other markings are superposed.

quite vanishes, but shrinks to about 200 miles in diameter and remains of this size for some three months by our reckoning. On the other hand, the maximum size of the southern cap (100° of latitude, or 3700 miles, measured along the planet's surface) is considerably greater than that of the northern cap (85°, or 3100 miles), and the southern cap begins to shrink a little earlier in the spring, and to appear again a little later in the autumn, than the other. These differences in behavior are obviously related to the fact that summer is hotter, and winter colder, in the southern hemisphere than in the northern hemisphere.

397. Other White Regions. The behavior of the polar caps immediately suggests that they are composed of *snow* — or, if not of frozen water, at least of some substance that melts or evaporates as soon as the temperature rises to a certain value in the spring, and that is carried to the other pole in the form of vapor by the planet's atmosphere and is precipitated there as soon as the surface grows cold enough. This theory is further confirmed by the occasional observation of white areas near the sunrise limb, which disappear as the planet's rotation carries them nearer the center of the disk, leaving the ordinary surface visible, — thus behaving exactly like deposits of hoar-frost, formed during the night and disappearing as the sun gets high.

The topography of the planet's surface has a definite influence on some of these phenomena. As the polar caps shrink, isolated white patches, which may last for days or weeks, are left behind in the same places every year. The last remaining portion of the southern cap is not at the pole, but about 7°, or 250 miles, from it, always in the same longitude (though the northern cap is very nearly centered on the pole). The winter "snows" too come down farther over certain definite regions (always reddish areas).

These facts suggest areas of high land, on which the "snow" lies longer than elsewhere. If Mars were proportionately as rough as the moon, the individual mountains would be easily visible at the terminator under favorable circumstances. Lowell estimates that any abrupt elevation much exceeding 2500 feet in height could thus be detected, and none have been found. But there seems to be no reason at all why gradual slopes (like that of the great plains from the Missouri to the foot of the Rocky

great interest but difficult to observe. Schiaparelli, in 1877 and 1879, announced the discovery of a great number of fine, dark, straight lines ("canals," as he called them) crossing the ruddy portions of the disk in all directions, and in 1881 he announced further that many of these became double at times, like the parallel tracks of a railway. W. H. Pickering, in 1892, added the detection of numerous small, dark spots connected with the canal system, and of darker markings within the dark areas, which Douglass, in 1894, described as canals similar to those in the reddish regions.

There is now no doubt regarding the real existence of these finer details, but the drawings and description of them by different observers are remarkably discordant.

(1) At one extreme stands Lowell, according to whom the canals, when well seen, are *very narrow* (from 15 to 20 miles wide at the most), *very dark*, *perfectly straight* (lying, with rare exceptions, along great circles on the planet's surface), and of *uniform width and intensity*, although under poor atmospheric conditions they may appear as hazy streaks. He found that they cover the planet's surface with a *complex network*, of *geometrical precision*, extending over both the ruddy and the darker regions, — four, six, or even as many as fourteen canals meeting exactly in one point, which is often marked by one of the dark spots, or *oases*, from 75 to 100 miles in diameter. More than 400 canals and nearly 200 oases have been observed and named at the Lowell Observatory. He found that some 50 of the canals are double, appearing as fine lines from 100 to 200 miles apart and equidistant throughout their whole length. At times only one of the components is strong enough to be visible. Fig. 142 represents typical drawings by Lowell.

(2) At the opposite extreme is Barnard, who, during years of observation with some of the greatest existing telescopes, never saw any trace of such a system of fine geometrical lines, although at times he saw "short, diffused, hazy lines, running between several of the small, very black spots which abound in this region" of the planet's surface, and "two long, hazy, parallel streamers." He said that, with the 60-inch reflector on Mt. Wilson, Mars gave "the impression of a globe whose entire

as fine, straight, sharp, dark lines. He never saw a canal double, though his assistant did, with the same telescope and on the same night, and a collaborator 500 miles away independently confirmed the assistant's observation.

All those observers who see the canals at all find them subject to great variations in visibility, which follow in a general way the similar changes in the dark areas. Lowell, from a study of several hundred of his drawings of the planets, reached very definite conclusions to the effect that the canals are faint or invisible during the Martian spring, and increase in prominence after the polar cap shrinks, — those nearest the pole darkening first, about the time of the summer solstice, while those in lower latitudes, with the oases connected with them, follow successively the "wave of quickening" which advances from latitude 70° to the equator in about 50 days, at the rate of some 50 miles per day, and continues into the opposite hemisphere for 1000 miles or more. Half a Martian year later, when this effect has very nearly faded out, a new "wave of quickening" starts from the opposite pole. Lowell also recorded the apparently capricious disappearance of certain canals for several years at a time, and the appearance of new canals where nothing had been seen for at least fifteen years and probably much longer.

400. All these accounts represent the mature judgment of trained and experienced observers after long and careful study of the planet under favorable conditions. To reconcile their extraordinary divergences is very difficult.

It is quite incredible that any one of them should never once have had the good fortune to see the planet under atmospheric conditions good enough to reveal its surface detail Some observers maintain that in such work a relatively small aperture is actually better than a very large one; but, however this may be in ordinary conditions of seeing, it is very improbable that it is true of the best moments, and conclusions based on the assumed inferiority of powerful instruments for such study should be regarded with great caution.

The only possible explanation appears to lie in a complex *personal equation*, on the part of the various observers, in seeing and recording these elusive markings, which, as all observers

and with the Lowell telescope is less than half this. In order to secure a larger image it is necessary to interpose an enlarging lens in the path of the rays. This magnifies the image but makes it correspondingly fainter, so that an exposure of several seconds is necessary even with very large apertures.

Under the best conditions beautiful photographs have been made, notably at Lowell, Yerkes, and Mt. Wilson observatories.

With respect to changes in the larger features of the surface the photographs are likely to have the final word, but with regard to the vexed question of the character of the canals they can give no decisive evidence. That faint markings of some sort are there they prove beyond question, but, owing to the relatively coarse grain of the plate, the best definition obtainable photographically is far below that which can be obtained visually with the same telescope; and, what is still more important, the eye can take advantage of the almost momentary flashes of good seeing, while the photograph records the average conditions during the seconds of exposure. Even though many successive exposures are made on the same plate, and those in which the definition is sharpest are picked out for study, the image of a narrow dark line and that of a diffuse gray band 100 miles or more in width would be identical in appearance on the best of them.

The photographs reproduced in Fig. 143 were made with yellow light. With red light the polar caps are less brilliant and the dark areas and canals are darker, while with green or blue light the opposite is true. On the photographs taken in violet light by Wright (Fig. 144), no detail at all is visible except the polar cap, which is brighter and larger than it is on photographs taken in light of greater wave-length. Except with the shorter wave-lengths there is little or no photographic evidence of the fading out of the surface features in a general luminosity near the limb, which is often noticed visually.

402. Atmosphere. There can be no doubt that Mars possesses some atmosphere, although probably a good deal less than the earth. The principal evidences of this are:

(1) *The behavior of the polar caps*, which can be explained only by the precipitation of some substance which has previously been in a state of vapor.

reflects light strongly enough to be visible is about 1/100 of the planet's radius, or 20 miles, as against 40 miles found in similar fashion in the case of the earth (§ 119). It is probable, however, that in the latter case the faint extension of the twilight can be followed a good deal farther than it can on Mars, and that the heights at which the atmospheres of the two planets have comparable densities are much more nearly alike.

Owing to the smaller force of gravity on Mars the density in the atmosphere would increase downward much less rapidly than on the earth; and if the densities were comparable in the two cases, at a height of, say, 20 miles, the Martian atmosphere would contain less material, exert a much lower pressure at the planet's surface, and yet rise higher in its rarefied upper layers, than the terrestrial atmosphere.

(3) *The partial obscuration of the surface markings by a general brightness at the limb.* This is exactly what may be expected if, near the limb, we are looking obliquely through a greater thickness of atmosphere, which reflects some light on its own account. The fact that this limb-light is shown on photographs only when the shorter wave-lengths are used indicates that it is blue scattered light like that of our sky, and also makes it probable that a part of the effect observed visually is a contrast effect arising from the proximity of the dark sky. The earth, similarly viewed, would undoubtedly show a much stronger limb-light.

(4) *The occasional appearance of clouds, fogs, or haze.* Though clouds are very rare on Mars in comparison with the earth, they have been observed many times. The edges of the polar caps, especially of the northern one when it is shrinking, have frequently appeared to be shrouded by a whitish veil, less brilliant than the cap itself. Several observers in recent years have reported rapid changes, almost from night to night, in the visibility of details over large areas of the surface of the planet, which seem explicable only by the formation or clearing of fog or haze. Most conclusive of all are the projections occasionally seen on the terminator, which are evidently the tops of high-lying clouds, catching the sun where all below is in shadow, — not mountains, for they last only a day or two and change their position in that short time, as measurements show.

403. Composition of this Atmosphere. The velocity of escape from the surface of Mars is 5.05 kilometers per second. It follows (§ 201) that the planet could retain an atmosphere of oxygen, nitrogen, and heavier gases, and probably water vapor as well, but not hydrogen or helium. The spectroscopic method (§ 619) is capable of detecting rather small quantities of oxygen and water vapor. All observers agree that the amounts of the latter in the Martian atmosphere must be smaller than in the earth's. The latest investigation (Adams and St. John, 1925) indicates that both are present and that "for equal areas the water vapor above the surface of Mars at the time of observation was of the order of 5 per cent, and the oxygen of the order of 15 per cent, of that normally in the earth's atmosphere."

How much nitrogen or carbon dioxide there may be can only be guessed, as they cannot be detected spectroscopically. The assumption that the amount of atmosphere above a square mile of the planet's surface is somewhere between one tenth and one half as much as on the earth appears to be consistent with the existing data. This would make the atmospheric pressure at the surface roughly between 4 per cent and 20 per cent of that at the earth's surface.

404. Temperature. The vexed question of the temperature of the planet's surface has apparently been conclusively settled by the radiometric observations (§ 618) of the planet made at the Lowell and Mt. Wilson observatories. The observers at both places, though using somewhat different methods, agree that the temperature of the planet's surface rises well above freezing in the equatorial regions at noon, and may reach 10° C. (50° F.) or even a little more. The dark areas are somewhat hotter than the reddish ones. Even at the equator the temperature is well below freezing at sunrise and sunset, and the nights must be very cold. The temperature of the polar cap appears to be as low as — 70° C., but after the southern cap has disappeared in late summer the surface becomes about as warm as at the equator.

405. Nature of the Polar Caps. These investigations appear to answer the long-discussed question of the nature of the polar caps and to make it very probable, to say the least, that they are actually composed of snow, — frozen water. The only

The dark regions may be regarded as vegetation, and the rest of the surface as desert. The enlargement and darkening of the areas as the moisture from the melting polar cap reaches them (whether in the form of streams, rain, or dew), and the changes in their color, toward green at this time and brown or gray in the dry season, are just what might be expected on this view; and minor differences in the course of these changes from year to year are not surprising.

An alternative theory suggested by Arrhenius supposes that the soil of the dark areas is saturated with soluble salts, which absorb moisture from the air, when this is available, and deliquesce, forming a darkish mud, but, when the atmosphere becomes very dry, effloresce, leaving a dry and light-colored surface. Such alkali flats are known in terrestrial deserts.

The presence of oxygen in the planet's atmosphere is strong evidence of the actual existence of vegetation on its surface, in the past at least (compare § 377). The attribution of the present seasonal changes to vegetation appears, therefore, to be decidedly the most reasonable hypothesis.

407. Concerning the nature of the canals, opinions differ even more widely. If the appearance of linear markings arises from the integration by the eye of details too delicate to be seen well (if at all) singly, no further explanation is required.

Arrhenius regards the canals as *cracks* or *fault-lines* in the planet's crust, along which the surface has been stained by escaping vapors, or as *rift-valleys* bounded by such faults, like the valley of the Dead Sea.

The supporters of the vegetation theory (the majority of the observers of Mars) generally follow W. H. Pickering's suggestion and regard the canals as *strips of vegetation bordering watercourses* crossing the arid regions, just as the valley of the Nile would appear to an observer on the moon like a green streak across the yellow African desert. As in this terrestrial example, the watered area may be very much wider than the watercourse, which by itself would be too narrow to be seen.

Seasonal changes in the visibility of the canals are then readily explicable; in fact, the Nile Valley would show just such changes after the annual flood.

(3) Though it is obviously useless to speculate concerning the physical organization or appearance of these inhabitants, it may be inferred, from the fact that the canals form a single system extending from pole to pole over the planet's whole surface, that they have established a world civilization embracing their race as a whole.

This theory has naturally aroused great popular interest, but it rests largely on conclusions regarding the surface details which are not generally accepted, and is far from being a necessary consequence of these.

Everything depends upon the geometrical character of the network of canals, and, as has already been pointed out, this may be the product of personal equation in the subconscious operation of the observer's mind. Again, the changes in the visibility of the canals may be explained in quite other ways; for example, as Lau has suggested, on the assumption that the planet's atmosphere becomes hazy in spring, concealing the finer details, and clears gradually in summer, beginning near the pole, so that the canals which have all already darkened under the haze come out successively in lower latitudes.

It is therefore necessary to render a verdict of "not proven" with regard to this theory. It should nevertheless be remembered that the development of a reasoned argument, showing how the existence of intelligent inhabitants on a planet fifty millions of miles away could be detected by such observations as can be made with existing telescopes, is in any event an admirable example of constructive scientific imagination.

409. Satellites. The planet has two satellites, discovered by Hall at Washington in 1877. They are extremely small and can be seen only with very large telescopes and when Mars is near opposition. The outer one, Deimos, is at a distance of 14,600 miles from the planet's center and has a period of $30^h\ 18^m$, while the inner one, Phobos, is at a distance of only 5826 miles and has a period of $7^h\ 39^m$, less than one third of the planet's day. (This is the only known case of a satellite with a period shorter than that of the rotation of its primary.) Phobos, therefore, *rises in the west*, as seen from the planet's surface, and *sets in the east*, completing its strange backward diurnal revolution

about 10 miles in diameter, and Deimos about 5 miles. On these assumptions Phobos, seen in the zenith from a point on the planet's surface directly beneath it, would appear about 1/3 the diameter of the full moon and 1/25 as bright, while Deimos would be 1/40 as bright as Phobos and only 80″ in diameter, and would appear to eyes like ours as a brilliant planet, like Venus.

According to Lowell neither satellite shares the red color of the planet.

THE ASTEROIDS

410. The asteroids, or minor planets, are a host of small bodies circulating around the sun between the orbits of Mars and Jupiter. The name "asteroid" (starlike) was suggested by Sir William Herschel early in the nineteenth century, as indicating that, though really planets, they appear like stars.

Kepler had noticed the wide gap between Mars and Jupiter and had tried to account for it, though unsuccessfully, and when Bode's law (§ 269) was stated, in 1772, the impression became very strong that there must be a missing planet in the vacant space, — an impression greatly strengthened by the discovery of Uranus in 1781, at a distance almost precisely corresponding to that law. The first discovery was made by the Sicilian astronomer Piazzi, who was then engaged in forming his extensive catalogue of stars.

On the first night of the nineteenth century (January 1, 1801) he observed a small star where there had been no star a few days earlier; the next day it had obviously moved, and it continued to move. He named the new planet *Ceres*, after the tutelary divinity of the island, and observed it carefully for several weeks, until he was taken ill; but before he recovered, the planet was lost in the evening twilight. It was rediscovered at the close of the next year by means of the calculations of Gauss, who invented, for the purpose, the method of determining a planetary orbit from three observations (§ 319).

In 1802 Pallas was discovered by Olbers while he was searching for Ceres. Juno was found by Harding in 1804, and in 1807 Olbers discovered Vesta, the only asteroid ever visible to the naked eye. The hunt for others was kept up for several years

amateur. While the telescope follows the stars by its driving-clock the plate-holder is given a slow motion, the same as that of an average asteroid in the region under observation. The image of an asteroid remains nearly stationary on the plate during the entire exposure, and is many times as intense as if it had been allowed to trail. The subsequent search is now for small black dots among a multitude of star-trails.

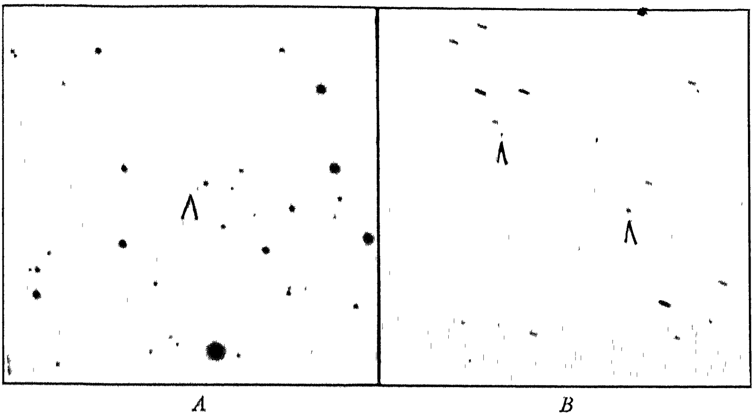


FIG. 146. The Asteroid (508) Princetonia

The photograph at the left was exposed for 182 minutes while the guiding star was kept carefully bisected by the cross-wires. The images of the stars are round, whereas that of the asteroid is a trail. (The cut was retouched to strengthen this; the trail was well shown on the original photograph.) Two asteroids appear on the photograph at the right. They are indicated by ink marks — as is customary. The plate was moved in the direction and with the speed calculated for *1911 NA*, which appears as a dot. The image of *Princetonia*, which had a different motion, is a short trail, and those of the stars are still longer trails. *Princetonia* was discovered and named by R. S. Dugan. (From photographs by Max Wolf, Königstuhl-Heidelberg)

When first announced, each asteroid is designated provisionally by the year followed by two letters, as 1924 RJ. The list of discoveries from 1893 to 1924 numbered 1726. These discoveries do not, however, refer to as many different objects. A number of the older asteroids have not been observed since the year of discovery, and are adrift and practically lost. Now and then they are picked up as new; thus, (132) Æthra was rediscovered in 1922 after having been lost since 1873. It is sometimes found, on calculating backward, that an asteroid whose orbit has

Curiously enough, the few asteroids whose periods are more than seven years seem to follow the opposite rule, and to cluster around the distances corresponding to periods $\frac{2}{3}$ and $\frac{3}{4}$ of Jupiter's or to Jupiter's period itself.

413. Other Orbital Elements. The inclinations average about 9° 30′, and the eccentricities about 0.15, — much greater than for the principal planets. The orbit of (944) Hidalgo is inclined 43° to the ecliptic; and that of (2) Pallas, nearly 35°. Of the first thousand, 3 others have inclinations exceeding 30°, and there are 17 more with inclinations between 25° and 30°. The eccentricity is also very large in some cases, — 0 53 for (887) Alinda, 0.54 for (719) Albert, and 0.65 for (944) Hidalgo (greater than for some comets), while it exceeds 0.30 in 25 other cases. There is a definite tendency for large eccentricity and large inclination to go together. The longitudes of perihelion again show the influence of Jupiter, nearly twice as many falling within 90° of Jupiter's perihelion as in the opposite half of the circle.

The orbits so cross and interlink that if they were material hoops or rings, the lifting up of one would take all the others with it, and those of Mars and Jupiter as well.

414. Diameter, Albedo, etc. These bodies are so small that micrometrical measurements, even of the largest ones, are very difficult, and of the smaller ones impossible. Barnard, with the Lick and Yerkes telescopes, has obtained measures of the disks of the four brightest and presumably largest, with the following results: Ceres, 480 miles; Pallas, 304 miles; Juno, 120 miles; Vesta, 240 miles. It is rather surprising that Vesta, which, if placed at the same distance, would look fully twice as bright as Pallas, and 20 per cent brighter than Ceres, should be so much smaller, but measures taken by Hamy, by an altogether different method (§ 824), confirm Barnard's value.

From these diameters, and from the observed brightness of the asteroids, it appears that Ceres, at the full phase, reflects nearly the same proportion of the incident light as does the moon; Pallas, about as much as Mars; and Vesta, almost as much as Venus.

The brightness of all four, and of all the other asteroids which have been photometrically observed (twenty or more in number),

so far been observable, and estimates are more uncertain; but from existing evidence it seems that 1/1000 of the earth's mass is a reasonable estimate, and 1/500 a liberal estimate, for the mass of all the asteroids, known and unknown.

Even a much larger amount of matter circulating between the orbits of Jupiter and Mars would be too small to produce perceptible perturbations of the orbit of Mars.

417. Form, Rotation, etc. In the case of such small bodies the gravitational forces, which compel a large planet to be nearly spherical in form, would be relatively inefficient, and it is not impossible that some of them may be of irregular shape.

A number of them show periodic fluctuations in brightness, such as might be caused by the rotation of such a body or of a spherical body covered with large bright and dark spots. Among these are (7) Iris, with a period of $6^h\,12^m$; (15) Eunomia, $3^h\,2^m$; (116) Sirona, $9^h\,40^m$; (345) Tercidina, $8^h\,47^m$; and (433) Eros, $5^h\,16^m$. It is natural to suppose that these are the rotation periods of the bodies.

This has been proved for Eunomia by Pickering and Wendell, who found that the apparent period of variation was longer when the planet was advancing than when it was retrograding. This is exactly what should happen if the rotation of the planet is direct; for, as can be seen from the general equations of synodic motion (§ 268), the apparent period should then be longer than the true period of rotation in the first case, and shorter in the second. They showed also that the variation was due to differences in albedo of opposite sides of the planet, and not mainly to irregularities of shape. In the latter case there would be two maxima of brightness in each rotation; in the former, but one. The observed change in period agreed with the former hypothesis.

418. The number of these bodies already known is so great, and the prospect of increase so indefinite, that it is a serious problem to take care of them.

Photographic methods now make it easy enough to observe all but the faintest asteroids; but to follow the motion of one of these little rocks by calculation is more troublesome, on account of the great perturbations produced by Jupiter, than to do the same for one of the great planets. Indeed, an exact solution of

great as the mean distance of Saturn. The orbit plane is highly inclined (43°.06), and the planet can never come anywhere near Saturn. But at its descending node its orbit comes within 26,000,000 miles of Jupiter. This suggests that it may have been thrown into its present extraordinary orbit by perturbations at a close approach to Jupiter (compare § 322). Its brightness corresponds to a diameter between 15 and 30 miles. At a perihelion opposition it is of about the eleventh magnitude; when in aphelion at opposition it is fainter than the nineteenth, so that it can only be observed at a favorable opposition.

The asteroid (719) Albert has a mean distance of 2.58 and an eccentricity of 0.54. At perihelion it can come within about 18,000,000 miles of the earth. Even at this distance it is only of about the twelfth magnitude. In aphelion it is of the twentieth magnitude and probably not observable with any existing telescope. It is probably only 2 or 3 miles in diameter.

420. Eros. This little planet, insignificant in size but of great astronomical interest, deserves special attention. It was discovered in August, 1898, photographically, by Witt, of Berlin, and at once attracted attention on account of its short period. Thanks to the numerous observations which have since been made, its elements are now known with very great precision. Its sidereal period is 643.23 days, or very nearly $1\frac{3}{4}$ years, and its synodic period 845 days, — the longest known.

Its mean distance from the sun is 1.458 times the earth's, or 135,430,000 miles; but the eccentricity of its orbit, 0.223, is so considerable that at aphelion it is 165,630,000 miles from the sun, well outside the orbit of Mars and within the asteroid region, while at perihelion its distance is 105,230,000 miles. The inclination of the orbit is 10° 49′, the perihelion within little more than 2° of the descending node and only 21° from the earth's perihelion, so that the least possible distance between the two planets is only *13,840,000 miles*. This is only a little more than half the least distance of Venus, and at such oppositions (which always happen about January 22) the parallax of Eros is nearly 60″.

Observations made on the planet at such time of close approach determine its parallax, and hence with the aid of our knowledge

421. Origin of the Asteroids. Two alternative but not mutually exclusive hypotheses have been advanced for the origin of this remarkable swarm of tiny planets. One is that they represent a planet spoiled in the making, that is, a mass of material which never coalesced to form a single body. The other and older theory is that they represent fragments of a planet which, for some reason, has exploded. No single explosion could account for the present tangle of orbits; for some of them are so much larger than others that, turn them about as one might, they could not be made to intersect. But this hypothesis has nevertheless been greatly strengthened by a remarkable recent discovery.

422. Families of Asteroids. If such a catastrophe should occur, and if the fragments were dispersed with relatively small velocities, their orbits would remain very similar, having nearly, though not quite, the same period, eccentricity, and inclination. At first they would all pass nearly through the point of explosion, but the perturbations, due mainly to Jupiter, would gradually shift the orbits, and after a few hundred thousand years this would no longer be true. It can be shown, however, that these perturbations would not alter, in the long run, the mean distances of the planets or their inclinations to the plane of Jupiter's orbit. Moreover, although the eccentricities and longitudes of perihelion would be altered by perturbations, they would change in such a manner that the centers of their orbits, when plotted in space, would all be equidistant from a certain definite point on the line joining the sun with the center of Jupiter's orbit.

K. Hirayama has shown that there exist several groups of asteroids which satisfy these conditions in a very striking manner. Five of these groups, containing from 15 to 44 asteroids apiece, have been recognized, and it seems impossible that the simultaneous agreement with all three conditions can be due to chance. It appears probable that the asteroids of these groups have actually been formed from a single central mass, although the details of the process remain obscure. A more violent explosion would produce orbits less like one another, and its effect would be much harder to trace, so that it is not at present known to what extent such processes may be responsible for the general run of the asteroids.

in very clear air, especially in the tropics, its extremities are seen to extend entirely around the ecliptic, forming a complete ring (the zodiacal band), on which, just opposite the sun, is a slightly brighter patch 10° or so in diameter, called the Gegenschein, or counterglow.

The zodiacal light, 30° or 40° from the sun, is very conspicuous, and it is probable that the regions near the sun, which are concealed from us by the twilight, are even brighter, though not bright enough to be seen through the diffused light which fills the air during even the longest total eclipse.

The spectrum of the zodiacal light has been photographed by Fath, who finds it identical with that of sunlight, as far as can be determined with the very low dispersion necessary in photographing so faint an object. Its light is partially polarized, as it would be if it were reflected, in part at least, from very fine particles, or molecules of gas.

Van Rhijn's work at Mt. Wilson indicates that the zodiacal light is not confined to the zodiacal belt but extends faintly over the whole heavens and accounts for nearly 60 per cent of the light of the sky on a moonless night. About 15 per cent more of the "sky light" originates in the earth's atmosphere and is apparently due to a faint permanent aurora (§ 658). If we could get rid of these two illuminations, the sky would be much darker and the Milky Way far more conspicuous.

The observations make it almost certain that the zodiacal light is *reflected sunlight* from innumerable small bodies, scattered throughout a region shaped like a lens or a much flattened ellipsoid of revolution, having its greatest diameter nearly in the plane of the ecliptic, and extending well beyond the orbit of the earth. Each individual particle, unless small enough to be held up by the sun's radiation pressure (§ 320), must be moving in its own independent orbit about the sun.

The brightening of the zodiacal band which forms the Gegenschein may be explained by the greater brightness of the individual particles at the full phase (as Searle suggests),[*] or (following Moulton) by the concentration of these particles which would take place, under the combined attractions of the earth and sun, in the neighborhood of a point on the line through these two

CHAPTER XII

THE MAJOR PLANETS

JUPITER · SATURN · URANUS · NEPTUNE

JUPITER

Jupiter, the nearest of the major planets, is usually next to Venus in order of brilliancy among the heavenly bodies. It is occasionally surpassed by Mars when that planet is nearest, but except when near conjunction it appears brighter than Sirius, the most brilliant of the stars. It is not, like Venus, confined to the twilight sky, but at the time of opposition dominates the heavens all night long.

425. Jupiter's orbit presents no marked peculiarities. The *mean distance* from the sun is 5.20 astronomical units (483,200,000 miles), and the *eccentricity* of the orbit is not quite 1/20, so that the distance from the sun varies about 47,000,000 miles between perihelion and aphelion.

At an average opposition the planet's distance from the earth is about 390,000,000 miles, while at conjunction the distance is about 576,000,000 miles; but it may come as near to us as 367,000,000 miles and may recede to a distance of nearly 600,000,000 miles.

The relative brightness of Jupiter at an average conjunction and at the nearest and most remote oppositions is respectively as the numbers 10, 27, and 18, and its corresponding stellar magnitudes are -1.4, -2.5, and -2.1.

The *sidereal* period is 11.86 years, and the *synodic* period is 399 days, a little more than a year and a month.

426. Dimensions. The planet's equatorial diameter, at mean distance, as found by Sampson from the durations of the eclipses of its satellites, is $37''.84$, corresponding to 88,640 miles. Measures of the disk with the filar micrometer give a value about $0''.75$, or 1700 miles, greater. The discrepancy is undoubtedly

quadrature, though at that time the edge farthest from the sun shows a slight darkening. The albedo is high, — 0.44 (Schönberg, 1921), — but not quite equal to that of Venus. The change of Jupiter's brightness with phase is small. Within the range of 12° of phase angle accessible from our point of observation on the earth the brightness (corrected for the variations in the distance) changes by about 10 per cent, while that of Mars, under similar circumstances, would change by 15 per cent, and that of the moon by 21 per cent.

It appears, therefore, that the surface of Jupiter must be much smoother than that of the other bodies.

The center of the disk is much brighter than the edge, — as is true in the case of the sun and Saturn, but not of Mercury, Mars, or the moon. The falling off in intensity is most rapid close to the limb, which, according to photometric measures by Schonberg, is only one eighth as bright as the center.

The contrast between the limb and the dark surrounding sky tends to obscure this effect in an ordinary telescopic view of the planet, but it is conspicuous in photographs and in visual observations in twilight. This darkening of the limb is readily explicable by the absorption of light in the atmosphere overlying a uniformly reflecting surface.

It has sometimes been supposed that the planet might be self-luminous to some extent, but this cannot be the case, for its satellites, when eclipsed by entering its shadow, become totally invisible.

429. Axial Rotation. Jupiter rotates on its axis more swiftly than any other planet, — in about $9^h\ 55^m$. The time can be given only approximately, not because it is difficult to find, and to observe with accuracy, well-defined objects on the disk, but because different results are obtained from different spots, according to their nature and their distances from the planet's equator. The rotation period is shortest near the equator; but in place of a gradual variation with the latitude, as in the case of the sun, there appear to be a number of different zones, rather sharply bounded and each with its own rate of rotation.

There is a great equatorial current, covering a zone from 10,000 to 15,000 miles wide, whose rotation period is a little more

Most of the markings are short-lived, lasting only a few weeks or months at most. From the manner in which they change their shapes and positions it is evident that they are atmospheric, like clouds.

431. Semi-Permanent Markings. There are, however, some markings which are at least semi-permanent and continue for

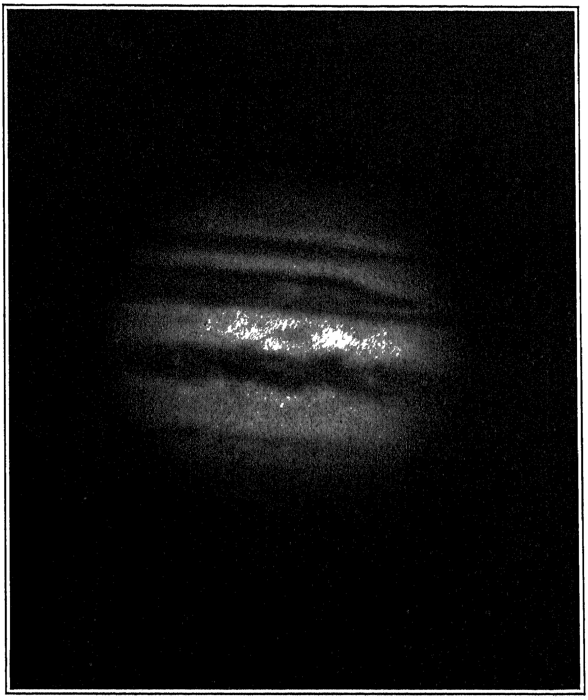


FIG. 148. Jupiter

October 19, 1915. The bay of the great red spot is seen near the right margin of the disk, above the middle. (From photograph by E. C Slipher, Lowell Observatory)

years with only slight changes. The most remarkable of these is the great red spot, shown in Fig. 148.

This was first noted in 1878, was extremely conspicuous for several years (being about 30,000 miles long by 7000 wide and brick-red in color), and then gradually faded away, losing its red color and slightly changing its form and becoming rounder. Even yet, while scarcely visible itself, the place which it occupies is clearly

432. Atmosphere and Spectrum. The spectrum of Jupiter is in general that of reflected sunlight, but there are strong bands in the orange and red which evidently arise from absorption in the planet's atmosphere.

They are diffuse in character (not resolvable into fine lines), and are evidently identical with similar and still stronger bands in the spectra of Saturn, Uranus, and Neptune (Fig. 208).

Their origin is still unknown, but their strength and character are sufficient to show that the atmosphere must be dense. The heaviest band of all, in the extreme red near λ 7200, is practically coincident with the strongest band of water vapor, but since the other bands of water vapor do not appear in the spectrum, Slipher, the discoverer of the band in question, believes that it must be due mainly to some other unknown constituent.

Spectroscopic observations upon the relative shift of the dark lines in the spectrum at the eastern and western limbs give a very fair determination of its rotation period.

433. Temperature and Physical Condition. The characteristics and rapid changes of the surface markings of Jupiter make it almost certain that they are clouds of some sort. The rapidity of their motions and transformations suggests that there is a vigorous circulation in the planet's atmosphere, with rapid exchange of material between the surface and the underlying depths. Until recently it was supposed that this demanded a fairly high temperature, and even that the surface might be almost red-hot, although not quite hot enough to be perceptibly self-luminous; but observations by Coblentz in 1914 and 1922 show that the radiation which comes to us from the planet is almost entirely reflected solar radiation, and indicate that the temperature of the surface is near $-140°$C., which is about what might be expected if very little heat comes up from the interior, so that the surface is warmed only by the sun's radiation. The atmosphere above the visible surface must therefore consist of the permanent gases, and the clouds may be of condensed particles of carbon dioxide or other substances which are familiar to us as gases, and which boil vigorously at temperatures far below zero.

The low mean density of Jupiter, as well as of the other major planets, presents a difficult problem of interpretation. We know that the inner portions are much denser, and the outer layers less dense, than the mean, and that there is a shell of atmosphere of unknown depth which is included in the measurement of the diameter of the planet. It could be explained on the assumption

has fairly well-defined gaseous banks, runs eastward at the rate of 250 miles an hour. Winds in the earth's upper atmosphere — also eastward — have often been observed to go half as fast.

434. Satellite System. Jupiter has nine satellites, so far as is known at present. Four of them (Fig. 151) are so large as to be easily seen with a common field-glass, and were, in a sense, the first heavenly bodies ever discovered, having been found by Galileo

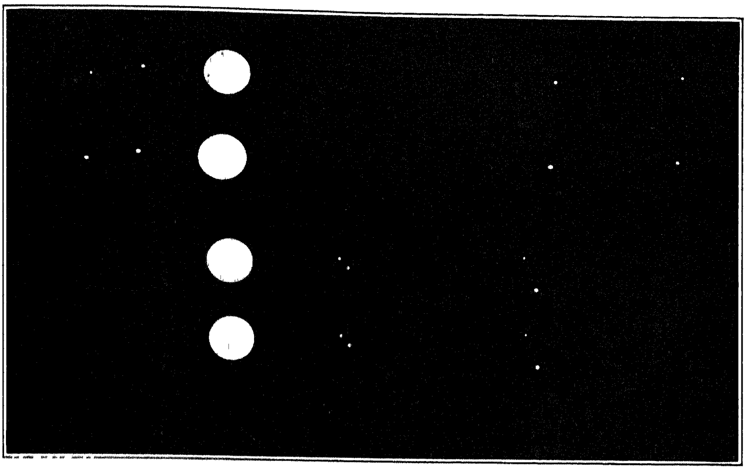


FIG. 151. Satellites of Jupiter

Photographed February 16, 1921 (top pair), and May 25, 1923, with the 24-inch refractor. The oblateness of the planet's disk is well shown. (From photographs by E. C. Slipher, Lowell Observatory)

in January, 1610, with the newly invented telescope. The others are exceedingly small and are visible only with very powerful telescopes. The fifth, the nearest of all to the planet, was discovered visually by Barnard in 1892. The remaining four have been found photographically since 1905, and are very remote from the planet. It is not improbable that other faint, distant satellites may remain to be discovered.

435. The Galilean Satellites. This name is often given to the four large satellites in honor of their discoverer, who, within a few weeks after his first observation, ascertained their true character and determined their periods with surprising accuracy. They are usually known as the first, second, etc., in the order of

The first and second satellites are therefore not far from the same size as our moon, while the other two are more than half as large again, — larger, indeed, than Mercury. With the eclipse diameters, which are probably the least affected by the systematic errors of observation, the densities of the four satellites come out 0.88, 0.87, 0 65, and 0.17 times the moon's density, or 2.9, 2.9, 2.2, and 0.6 times that of water. It is therefore probable that the first two are masses of rock, like our own satellite Jeffreys suggests that the third and fourth may be composed largely of ice or solid carbon dioxide. The velocity of escape from the surfaces of all four is about the same as that from the moon, so that, unless they have always been very cold throughout their history, they can hardly possess atmospheres. Slipher finds that the atmospheric band, conspicuous in the spectrum of Jupiter, is absent from that of the third satellite.

438. Brightness and Albedo. The third satellite is the brightest as well as the largest and most massive. The first and second are nearly equal in brightness, and are about two thirds as bright as the third, while the fourth is little more than half as bright as these two.

All four would be visible to the naked eye on a clear, dark night if they were not so near the planet, and the third would be an easy object, like a star of the fifth magnitude. But since the third satellite is never more than 6′, nor the fourth more than 11′, from Jupiter, which is more than 800 times as bright as the brighter of the two, they cannot be seen without optical aid, except perhaps by extraordinarily keen eyes, under very favorable conditions.

The albedo of the first satellite appears to be very nearly equal to that of Jupiter; that of the second, greater by about 20 per cent than the average for the planet's disk; that of the third, about as much less; while that of the fourth is less than one third that of Jupiter. These values are confirmed by the appearance of the satellites when in transit in front of the planet. The second always shows bright; the first, bright except against the brightest parts of the planet's disk; the third, darkish except when near the limb, and the fourth, grayish even near the limb, and almost black at mid-transit.

As seen from the planet's surface the first satellite would give about one fifth as much light as our moon; and the other three

revolution (Fig. 152). At conjunction they cast their own shadows upon the planet, and these shadows can easily be seen in the telescope as black dots on the planet's disk (Fig. 153), — *shadow-transits*. The satellites themselves, which *transit* the disk about the same time, are much more difficult to observe.

When the planet is exactly in opposition, the shadow, of course, is directly behind it, and we observe an *occultation*

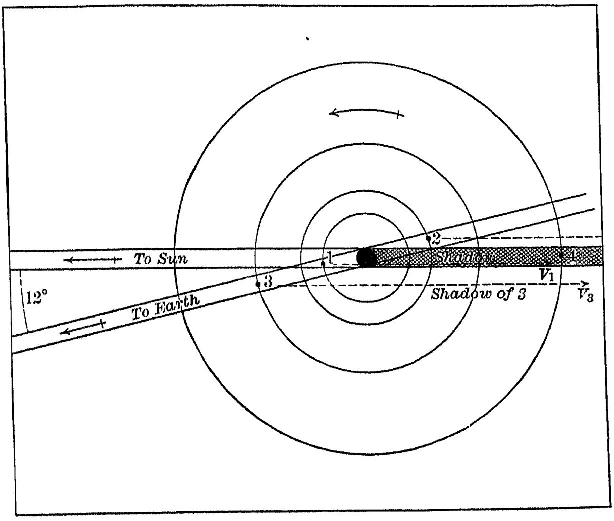


FIG. 152. Eclipses, Transits, and Occultations of Jupiter's Satellites

The planet is supposed to be at western quadrature. The shadow of the planet projects so far to one side that the whole eclipse of satellites II, III, and IV takes place clear of Jupiter's disk. Satellite I is in transit, preceded by its shadow, II is occulted; III is in transit, and IV is in eclipse

(§ 244) instead of an eclipse. At other times we ordinarily see only the beginning or the end of eclipse; but when the planet is at or near quadrature, the shadow projects so far to one side that the whole eclipse of every satellite except the first takes place clear of the disk.

The times of occurrence of all the phenomena of the satellites — eclipses, occultations behind the planet, and transits of the satellites and their shadows across its disk — are given in the *Nautical Almanac* each year. These four types of phenomena provide very interesting observations for even a small telescope.

The moment during these gradual changes at which the satellite is last seen (or first glimpsed when coming out of the shadow) depends on the state of the air and of the observer's eye, and on the power of his telescope; so that observations of the time of disappearance or reappearance sometimes differ by a minute or two in the case of the first satellite, and by five or even ten minutes for the fourth, making them practically useless for the determination of longitude (§ 107). More precise results can be obtained by measuring the brightness of the satellite with a photometer, using one of the other satellites as a standard of comparison. From a series of such observations, twenty or more of which can sometimes be made during one eclipse, the moment when the satellite is of just half its normal brightness (that is, when it is just half immersed in the planet's shadow) can be determined within a very few seconds.

A very large number of such observations of the eclipses of all four satellites have been made at Harvard by Professor Pickering and others. They are of great value both in determining the exact elements of the orbits and motions of the satellites and for finding the diameters of the satellites and of Jupiter, and have formed the observational basis for Sampson's new tables, by which their motions may be accurately predicted

Sampson finds that the eclipses sometimes come early or late. He suggests that the diameter of Jupiter is variable, the clouds in its atmosphere lying higher at some times and places than at others, and ranging as much as 100 miles on either side of their mean level. Some such explanation seems to be demanded by the facts.

442. The Equation of Light. In 1675 Roemer, a Danish astronomer (the inventor of the transit instrument, meridian circle, and prime-vertical instrument, — a man almost a century in advance of his day), found that the eclipses of Jupiter's satellites show a peculiar variation in their times of occurrence, which he explained as due to the *time taken by light to pass through space.* His bold and original suggestion was neglected for more than fifty years, until long after his death, when Bradley's discovery of *aberration* (§ 162) proved the correctness of his views.

When we observe a celestial body, we see it, not as it *is* at the moment of observation, but as it *was* at the moment when the

very good conditions. Its distance from the planet's center is 112,600 miles, and its period, $11^h\ 57^m\ 22^s.7$. Its orbit is very nearly circular and almost in the plane of the planet's equator, but the eccentricity (0.003) and inclination to this plane (27′) have been detected by careful observations. The influence of the planet's ellipticity causes the line of apsides to advance, and the nodes to retrograde at the enormous rate of 916°, or more than $2\frac{1}{2}$ complete revolutions per year (according to H. Struve). The ellipticity of the planet can be very accurately determined from these motions.

The satellite itself is far too small to show a disk, and its diameter can be estimated only from its brightness (which has never been accurately measured). If of the albedo of the first satellite, it would be about 75 miles in diameter; but if similar to the fourth, 150 miles. The first figure is perhaps the more probable.

444. The Outer Satellites. The sixth and seventh satellites (in order of discovery) were found by Perrine at the Lick Observatory, in December, 1904, and January, 1905, on photographs made with the Crossley reflector. The sixth, which is the brighter of the two, is of magnitude 13.7 (according to visual estimates by Barnard) and is probably about 100 miles in diameter. The seventh is much fainter and may be 40 miles in diameter.

The periods of these two satellites are nearly equal (250 and 260 days), and their mean distances from the planet are 7,100,000 and 7,300,000 miles. Both their orbits have large eccentricities (0.15 and 0.21), and their planes are inclined 29° and 28° to that of Jupiter's orbit, but in different directions, so that they make an angle of about 28° with one another. The two orbits, though nearly of the same size, interlock like two links of a chain, and nowhere come within 1,800,000 miles of one another.

Both satellites are subject to very large solar perturbations which may affect their longitudes by several degrees in either direction.

The eighth satellite was discovered photographically by Melotte at Greenwich in February, 1908. It is a very faint object, about equal to the seventh satellite in brightness and, presumably, in size.

the eighth, and may be 25 miles in diameter. Its motion is retrograde, like that of the eighth satellite, and its mean distance from the planet a little greater (15,000,000 miles), with a period of two years and two months, eccentricity 0.25, and inclination 156°, according to the elements derived from the observations of 1914 and 1915. Its perturbations are also very large.

The average apparent distances of the eighth and ninth satellites at elongation exceed 2° (when Jupiter is near opposition), and under favorable circumstances they may be greater; thus the eighth satellite was 2° 47′ from the planet in July, 1913. When these satellites have been observed through half a dozen revolutions, a study of their motions will give a very precise determination of the mass of Jupiter (and will present a laborious and difficult problem to the mathematical investigator).

445. Are these Satellites Captured Asteroids? If, in the course of the changes of its orbit, the eighth satellite should ever get about twice as far from Jupiter as it did in 1913, and should do this near the time of its opposition or conjunction with the sun, the sun's disturbing force upon it would equal the whole attraction of the planet and counteract it almost completely, so that the satellite would move away out of the range of the planet's attraction altogether, and circulate in an independent orbit of its own about the sun, becoming a faint asteroid. It is equally conceivable that, reckoning backwards, the reverse process may have occurred at some past time, — an asteroid approaching the planet so slowly and in such a manner that under the combined attraction of the planet and sun (the planet's attraction alone could not do it) it was "captured," and settled down to move around the planet as a satellite

The question whether any of the distant satellites of the planets have thus been captured (and are therefore likely to be lost again in the future) is of great interest. The problem has been solved mathematically in the case where the mass of the satellite is negligible and the planet's orbit circular.

Calculation shows that if the planetary orbits were circular, the moon, Jupiter's sixth and seventh satellites, and the ninth satellite of Saturn (soon to be described) would be rigorously confined to closed regions surrounding their respective primaries. Since the actual orbits of the planets are elliptical, the calculations do not *prove* that this is actually the case; but the "margin of safety" is in all cases great enough to make it exceedingly probable that these satellites have been circulating about the planets for an indefinite period, and will continue to do so indefinitely.

For the eighth and ninth satellites of Jupiter the calculations show that no such necessary restrictions exist; but, with their retrograde motion, Moulton considers it reasonable to believe that their motions are stable.

uncertain by 1 per cent, and the surface and volume by 2 and 3 per cent respectively.

449. Mass, Density, and Gravity. The planet's mass is well determined both by the observations of its satellites and by the perturbations which it produces upon Jupiter, and may be taken as 1/3499 that of the sun, or 94.9 times the earth's mass, with a probable error of perhaps one part in a thousand.

The remarkable fact follows that its mean density is only 0.13 times that of the earth, or *0.715 times the density of water*. It is by far the least dense of all the planets. The superficial gravity averages 1.17 times that at the earth's surface, but varies by 30 per cent between the equator and the pole.

450. Axial Rotation. The surface of Saturn is marked by belts, parallel to its equator, as in the case of Jupiter, but it is only very rarely that spots appear which are well enough defined to permit a determination of the rotation period. A white spot near the equator, which suddenly appeared in 1876 and continued visible for several weeks, gave a period of $10^h\ 14^m\ 24^s$, according to Hall, its discoverer. One discovered by Barnard in 1903, in latitude 36° north of the planet's equator, gave the much longer period of $10^h\ 38^m$. It is evident that on Saturn, as on Jupiter, the rotation periods must be different in different latitudes, and it is significant that in this case too the rotation is most rapid near the equator.

The centrifugal force at the equator is 0.17 of the force of gravity. The ratio of the planet's ellipticity to this quantity is only 0.62, — so near to the theoretical limit (0.5) for a body whose whole mass is concentrated at the center as to make it certain that the superficial layers must be of very low density, much of the mass being probably concentrated into a deep-lying nucleus.

The position of the equatorial plane is accurately determined by the perturbations which the planet's ellipticity produces in the planes of the satellites' orbits. It is inclined 28° 6′ to the plane of the ecliptic, or 26° 45′ to that of the planet's orbit.

451. Surface Markings, Color, and Spectrum. As in the case of Jupiter, the edges of the disk are much less brilliant than the center. The belts are less sharply defined and less variable than those of Jupiter. There is usually a brilliant yellowish zone at the equator, and a darkish cap, of greenish hue, at the pole.

The planet's spectrum shows bands in the orange and red, similar to those in Jupiter's spectrum, but stronger. These bands are absent in the spectrum of the ring, which presumably has no atmosphere.

452. Brightness and Albedo. Saturn appears to the naked eye to be comparable in brightness to the brighter fixed stars. In addition to the changes in its apparent brightness arising from the variations due to its distance from the earth and sun, there are large variations due to the different aspects under which we see its rings. When these are edgewise toward the earth, and practically invisible, the planet in opposition appears like a star of magnitude 0.9, — about as bright as Altair, — and in conjunction one third fainter. But when the rings are most widely displayed they reflect to us about one and two-thirds times as much light as the ball of Saturn, and its brightness is correspondingly increased. This phase occurs near the planet's perihelion and aphelion. At opposition in the former case it appears of magnitude, − 0.4, — brighter than any star visible in the latitude of New York, except Sirius, and $3\frac{1}{2}$ times as bright as at opposition with "ring invisible." At aphelion it is about one third fainter.

The albedo of the ball of Saturn is 0.42 (according to Schönberg), — a little less than that of Jupiter, and suggesting a similar physical condition. The photographic albedo is 25 per cent less, in good accordance with the yellowish color which the planet presents to the eye.

When the ring is invisible, the planet's light shows a small change with phase, which indicates that its reflecting surface, like Jupiter's, is effectively a smooth one. When a large part of the light comes from the rings, however, much greater variation with phase is revealed by the observations. This leads to important conclusions regarding the nature of the rings (§ 459).

453. The physical constitution of the planet presents problems similar to those offered by Jupiter, but more acute. There can be no doubt that the visible surface is gaseous and that the atmosphere is very deep, but how the mean density of the interior can be so low is not yet fully understood. Coblentz's observations indicate that the temperature of the surface is about − 150° C. Even this is some 30° higher than the sun's radiation could keep it.

or "gauze" ring between the principal ring and the planet. (It was discovered a fortnight later, and independently, by Dawes in England.)

455. Dimensions of the Rings. The outer ring, *A*, has an exterior diameter of 171,000 miles, according to Lowell's recent measures, and is a little more than 10,000 miles wide. "Cassini's division" between it and ring *B* is probably about 3000 miles wide (although, owing to irradiation, most measures make it narrower) and appears to be perfectly uniform all around. Ring *B* is 145,000 miles in outer diameter, and a little less than 16,000 in width. It is much brighter than *A*, especially at its outer edge; indeed, as Fig. 156 shows, it is as bright as the brightest parts of the planet's surface. According to Lowell it is separated by a narrow gap, perhaps 1000 miles wide, from ring *C*, which is sometimes known as the crape ring, because it is only feebly luminous and is semi-transparent, allowing the edge of the planet to be seen through it. This innermost ring is about 11,500 miles wide, making the width of the whole ring system 41,500 miles, and leaving a clear space of 7000 miles between its inner edge and the planet's equator. The thickness of the rings is very small indeed, probably not exceeding 10 miles. If we were to construct a model of them on the scale of 10,000 miles to an inch, so that the outer one was fully 17 inches in diameter, they would be thinner than the thinnest tissue paper. This extreme thinness is proved by the appearance presented when the plane of the ring is directed toward the earth, as it is once in every fifteen years. At that time the ring becomes invisible for several days even in the most powerful telescopes.

456. The outer ring, *A*, is occasionally seen divided by a narrow dark line known as "Encke's division," but more usually there is only a darkish streak upon it.

Lowell has seen and measured several divisions in ring *B*, — fine linear markings, concentric with the ring. No markings, however, which would permit a determination of the rotation period of the rings have ever been observed.

The shadow of the planet on the rings is of course conspicuous, except at opposition, when it is concealed behind the planet. That of the ring on the planet, also, can frequently be seen as a narrow dark band bordering the ring where it crosses the disk.

once or three times, according to circumstances. In the former case the disappearance of the rings occurs when the planet is near conjunction, under unfavorable conditions for observation, as in 1878 and 1891. In the latter, two at least of the three disappearances are well observable, as was the case in 1921.

458. Appearance of the Dark Side of the Rings. When the rings are exactly edgewise toward us, they are invisible for a day or two even with the great Yerkes telescope; but when the earth and sun are on opposite sides of the plane of the rings, so that we see the dark side of the rings (greatly foreshortened, of

South

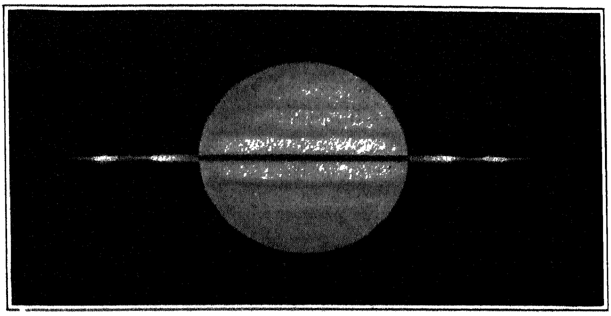


FIG. 159. Saturn

As seen with the 40 inch refractor of the Yerkes Observatory, December 12, 1907, when the earth and the sun were on opposite sides of the plane of the rings. (From drawing by E. E. Barnard)

course), they are visible, and present a very remarkable appearance, first described by Bond in 1849 and observed in great detail by Barnard and others in 1907 (Fig. 159).

The whole surface of the rings is visible, though very much less brightly illuminated than the planet's disk, so that where the rings cross the disk they appear as a narrow dark band. On the faint hazy strip outside the planet appear two brighter regions, or condensations, on each side, which are brighter but, according to Barnard, no wider than the remainder of the rings. These are permanent phenomena, having been seen by many observers throughout the time when the dark side of the rings was visible. Measures of their distances from the planet show that the inner ones coincide in position with the crape ring, while the outer ones

sun's rays traversed the ring very obliquely, at an angle of 11°. If they had gone through squarely, the loss of light would have been less than one fourth as great, whence it appears that if the crape ring could be viewed at right angles to its own plane, the solid particles, even in its densest portion, would cover only about one eighth of the background. Ring B, which has an apparent albedo fully as great as that of the planet, must be much more closely packed and is probably quite opaque. Ring A, which is fainter, is partially transparent. This was proved in 1917 by the observations of Ainslie and Knight, who watched a star of the seventh magnitude pass behind the outer ring and the Cassini division. Behind the outer ring the star was much fainter than normal, but it brightened up twice for a few seconds, presumably while it was crossing two of the narrow divisions sometimes observed in ring A. The star apparently shone with its full brightness while crossing the Cassini division.

(2) *Photometric observations of the planet's brightness.* It has already been mentioned that the brightness of the light reflected from the rings falls off considerably as the phase angle increases, though that of the planet's ball does not. When the rings are widely open, the combined light at quadrature (phase angle 6°) is 79 per cent of that at opposition (after correction for the effect of changes in distance). This shows that the light from the rings, which forms rather more than half of the whole, has diminished by 35 per cent.

Seeliger has shown that this is a direct consequence of the meteoric constitution of the rings. At different points our line of sight penetrates to very different depths between the particles of which the ring is composed, — rarely, however, getting clear through without striking something. The sun's rays do the same, and the particles lying nearest the sunlit side of the rings cast shadows on those which lie deeper. When the earth is exactly in line between the sun and Saturn, each particle *hides its own shadow*, and the whole surface of the rings appears bright. But when we are even a degree or two out of line, the shadow of each particle begins to come out from behind it, and the multitude of tiny shadows diminishes the total light. From the observed behavior Seeliger concluded that, on the average, the particles

of the rings, — which are very similar to those arising from the ellipticity of the planet, but fall off more rapidly with distance.

It is extremely probable that the rings, as a whole, are stable and form a permanent feature of the Saturnian system. The finer divisions may, and probably do, change somewhat from time to time.

Kirkwood has shown that the divisions in the rings are very probably due to the perturbations produced by the satellites, since they occur at distances from the planets where the period of a small body would be precisely commensurable with that of some one of the inner satellites. Thus, the distances at which the period would be 1/2 that of Mimas, the innermost satellite, and 1/3 and 1/4 of those of the next two satellites, all fall within Cassini's division; the boundary between rings *B* and *C* corresponds to 1/3 of the period of Mimas, Encke's division to 3/5 of it, and the divisions recently observed by Lowell to various other values, — 2/5, 3/8, and so on.

It will be remembered that there are similar gaps in the distribution of the asteroids, in places where their periods would be commensurable with that of Jupiter. In both cases the gaps are nearly but not quite bare of small bodies.

In 1850 Roche proved that a liquid satellite of any planet, if at a distance greater than a certain definite limit, would be merely distorted by the tide-raising forces resulting from the planet's attraction; but that if it were nearer than this limit, these forces would overcome the mutual gravitation of its parts, and actually tear the satellite to pieces. For a satellite of the same density as the planet this limit is 2.44 times the planet's radius. Now the distance of Mimas, the nearest satellite, is 3.11 times Saturn's radius, so that it lies well outside Roche's limit, while the outer radius of ring *A* is 2.30 times that of the planet, placing the whole ring system in the region in which any satellite-forming material would, if liquid, have been torn to bits by the tidal forces, and would, if composed of separate solid particles, have been prevented by these forces from aggregating into larger masses.

462. Satellites. Saturn has nine of these attendants, the largest of which, named Titan, was discovered by Huygens in 1655. It is easily seen with a 3-inch telescope. D. Cassini,

is the tiny retrograde satellite Phœbe, at a mean distance of 8,034,000 miles, and with a period of 550^d.

The five inner satellites move in nearly circular orbits, and nearly in the plane of the rings; the orbit of Mimas is inclined $1\frac{1}{2}°$ to that plane, and that of Tethys 1°. The orbit planes of Titan and Hyperion deviate slightly from the plane of the rings, in the direction of the orbit-plane of the planet; and that of Iapetus is not far from halfway between the two.

Phœbe, the faint and distant satellite which was discovered by W. H. Pickering in 1898, has a *retrograde motion*, unprecedented at the time of its discovery, but later matched by the eighth and ninth satellites of Jupiter. The orbital eccentricity (0.17) and inclination (5°.3, or, more properly, 174° 7) are considerable, but not so great as in the case of Jupiter's outer satellites.

Pickering, in 1905, reported the discovery of another very faint satellite with a period of 20 85 days, which he named Themis, but the discovery has not been confirmed. It may be that one or more faint satellites of about that period exist.

463. Diameters, Albedo, Rotation. Titan shows a distinct disk in large telescopes, and the measures of Barnard and Lowell make its diameter, at mean distance, about 0″.60, or 2600 miles. Its brightness is a little more than 1/1000 that of the planet (not including the rings), from which it follows that its albedo is about four fifths of the average for the planet's disk. This agrees well with observations of its transits by Struve, who finds that near the middle of the disk Titan appears as a dark spot, while it is indistinguishable near the limb.

The remaining satellites are too small to show perceptible disks. Rhea, the brightest after Titan, takes about four minutes to enter or leave the planet's shadow during an eclipse, which shows that its diameter must be about 1100 miles and its albedo about equal to that of the planet. Its shadow on the planet has been seen at the Lowell Observatory.

The diameters of the minor satellites can be estimated only from their measured brightness. If they are comparable in albedo with the planet and the denser part of the rings (as seems probable), Dione and Tethys must be 700 or 800 miles in diameter, Enceladus about 500, and Mimas perhaps 400.

URANUS

465. Discovery of Uranus. Uranus was the first *planet* ever "discovered," and the discovery created great excitement and brought high honors to the astronomer. It was found accidentally by the elder Herschel on March 13, 1781, while "sweeping" the heavens for interesting objects with a 7-inch reflector of his own construction. He recognized it at once by its disk as something different from a star, but, never dreaming of a new planet, supposed it to be a peculiar kind of comet; its planetary character was not demonstrated until nearly a year had passed, when Lexell showed by his calculations that it was doubtless a planet beyond Saturn, moving in a nearly circular orbit.

It is distinctly visible to a good eye on a dark night as a faint star, almost exactly of the sixth magnitude, but is very far from being conspicuous.

The name "Uranus," suggested by Bode, finally prevailed over other appellations that were proposed (Herschel had called it Georgium Sidus, in honor of the king).

It was found, on reckoning backward, that the planet had been many times observed as a star and had barely missed discovery on several previous occasions. Twelve observations of it had been made by Lemonnier alone, and later they proved extremely valuable in connection with the investigations which led to the discovery of Neptune.

466. Orbit. The mean distance of Uranus from the sun is 19.19 astronomical units, or 1,782,300,000 miles. The actual distance varies 84,000,000 miles on each side of this, owing to the eccentricity of the orbit (0.047). The inclination of the orbit plane to the ecliptic is small, only 46′. The planet's periodic time is 84.01 years, and the synodic period 369.16 days. The orbital velocity is $4\frac{1}{4}$ miles per second.

It is so remote that its apparent brightness varies by only 20 per cent from opposition to conjunction, and there is no perceptible difference in its appearance at opposition and quadrature.

467. Dimensions, Mass, Density. Uranus, under sufficient telescopic power, shows a sea-green disk about 3″.75 in diameter (at mean distance), corresponding to a real diameter of some

469. Rotation. The planet's ellipticity shows that it must be in rapid rotation, but the period remained undetermined until 1912, when Lowell and Slipher, by the spectroscopic method (compare § 579) derived the value *$10\frac{3}{4}$ hours*. Though the planet's spectrum was necessarily narrow, the inclination of the lines was unmistakable, and the period given, though possibly uncertain by half an hour or more, must be substantially correct. The *direction of rotation* agrees with that of the motion of the satellites, and the equatorial plane must coincide almost exactly with that of their orbits, for otherwise the planes of their orbits would undergo considerable perturbations, and such have not been observed.

A striking confirmation of the spectroscopic period was furnished in 1917 by the photometric observations of Leon Campbell, which showed a variation in the brightness of Uranus amounting to 0.15 magnitude in a period of 10 hours and 49 minutes. It is probable that an accurate value of the period of rotation can be obtained by continuing the observations.

Several observers have reported extremely faint bands or belts upon the planet, much like the belts of Jupiter seen with a very small telescope. The earlier observers found that these belts were inclined to the satellites' orbit plane at a considerable angle (though they were so faint that it was difficult to be sure about it), but Lowell's later observations make the directions of the two practically coincident. No distinct markings, permitting a direct determination of the rotation period, have ever been observed.

470. Satellites. The planet has four satellites, — Ariel, Umbriel, Titania, and Oberon, — Ariel being the nearest to the planet. The two brightest, Oberon and Titania, were discovered a few years after the discovery of the planet, by Sir William Herschel, who thought he had discovered four others also; he may have glimpsed Ariel and Umbriel, but it is very doubtful. They were first certainly discovered and observed by Lassell in 1851.

These satellites are telescopically the faintest bodies in the solar system and the most difficult to see (excepting some of the recently discovered satellites of Jupiter and Saturn). Oberon and Titania are of about the fourteenth magnitude, the latter being a little the brighter, and they can be just glimpsed under the best conditions, with a telescope of ten or twelve inches

appeared perfectly round, while the former epochs were best for determining the inclination of the orbits, the position of the nodes, and the polar compression of the planet.

NEPTUNE

472. Discovery of Neptune. This is reckoned as the greatest triumph of mathematical astronomy since the days of Newton. After Uranus was discovered, it was very soon found impossible to reconcile the old observations of that planet, by Lemonnier and others, with any orbit that would fit the observations made in the early part of the nineteenth century; and, what was worse, the planet almost immediately began to deviate from the orbit computed from the new observations, even after allowing for the disturbances due to Saturn and Jupiter. It was misguided by some unknown influence to an amount almost perceptible by the naked eye; the difference between the actual and computed places of the planet amounted, in 1845, to the "intolerable quantity" of nearly two minutes of arc.

By modifying the elements of the planet's orbit, on which the computations were based, it proved possible to represent the observations from 1782 to 1845 with errors nowhere exceeding 20″. One might think that such a discrepancy between observation and theory was hardly worth minding; but the way in which observations by different astronomers, in successive years, agreed with one another and disagreed with theory made it certain that some unknown force must be acting on the planet. In exact science such unexplained but unquestionably real "residuals" are often the seeds from which new knowledge springs, and in this case the discrepancies supplied the data which sufficed to determine the position of a great world, theretofore unknown.

As a result of a most skillful and laborious investigation, Leverrier, a young French astronomer, wrote in substance to Galle, then an assistant in the Observatory at Berlin:

"*Direct your telescope to a point on the ecliptic in the constellation of Aquarius, in longitude 326°, and you will find within a degree of that place a new planet, looking like a star of about the ninth magnitude, and having a perceptible disk.*"

473. The Planet and its Orbit. The planet's mean distance from the sun is 30.07 astronomical units, or 2,792,700,000 miles (instead of 3,600,000,000 as predicted by Bode's law). The orbit is very nearly circular, its *eccentricity* being only 0.0086. Even this, however, corresponds to a variation of 48,000,000 miles in the planet's distance from the sun. The *period* is 164.8 years (instead of 217, as indicated by Leverrier's computed orbit), and the *orbital velocity* is about $3\frac{1}{3}$ miles per second. The *inclination* of the orbit is 1° 47′.

Neptune is invisible to the naked eye but can easily be seen with a good field-glass, appearing as a faint star of magnitude 7.7. With a small telescope it can only be distinguished from the neighboring stars by means of its motion from night to night, but with larger instruments it shows a greenish disk a little more than 2″ in diameter. As in the case of Uranus the older measures made the diameter too large. Abetti, from a discussion of all the observations up to 1912, concludes that the diameter is 2″.3, corresponding to 31,000 miles, or 3.92 times the earth's diameter.

The planet's mass is known with far greater percentage accuracy, both from its satellite and from its action on Uranus, and is 1/19,350 of the sun's mass, or 17.16 times that of the earth. With the diameter given above, the density comes out 0.29 times the earth's, or 1.6 times that of water, and the superficial gravity 1.12 times our own. The albedo, calculated from the observed brightness and this diameter, is 0.52, — a high value, and somewhat uncertain on account of the difficulty of measuring the diameter.

The *spectrum* shows the same bands which appear in the case of the other major planets, but much stronger (Fig. 208), indicating a very extensive atmosphere.

Menzel has suggested that the unknown substance responsible for these bands may be some compound which is completely decomposed at ordinary temperatures and can exist in quantity only where the atmosphere is very cold. This would account for the steady increase in the strength of the bands from Jupiter to Neptune, and might perhaps also explain why they have not been found in the laboratory.

The planet's disk is sensibly circular and shows no markings, so that there is no direct evidence concerning its rotation. It will be noticed that Uranus and Neptune form a "pair of twins,"

be the plane of the planet's equator (§ 342). From the observations so far available it appears that the satellite's orbit is inclined about 20° to the planet's equator, and the latter about 29° to Neptune's orbit, while a revolution of the nodes takes about 580 years (Eichelberger and Newton, 1926).

From these data it is possible to calculate the planet's oblateness, and its period of rotation, by making various assumptions regarding the ratio of the ellipticity to the centrifugal force at the equator This ratio depends on the planet's internal constitution (§ 341).

If the internal constitution is assumed to be like that of Saturn, the oblateness comes out 1/32 and the rotation period 12 hours, if it is like Jupiter's, the results are 1/57 and $17\frac{1}{2}$ hours, if it is like the earth's, the results are 1/81 and $23\frac{1}{2}$ hours. The truth probably lies within this range and nearest to the middle set of values, and it is almost certain that Neptune rotates more slowly than the other three major planets and faster than the earth. The calculated ellipticity would be imperceptible in so small a disk, but the matter may soon be tested spectroscopically, as in the case of Uranus.

475. The Solar System as seen from Neptune. At Neptune's distance the sun itself would have an apparent diameter of a little more than 1′ of arc, — only about the diameter of Venus when it is nearest to us, and too small to be seen as a disk by the unaided eye (if there were eyes like ours on Neptune). The light and heat received from the sun by Neptune are only 1/900 of what the earth receives. Even so, the intensity of sunlight at Neptune would be 520 times that of full moonlight on the earth, or equal to that of a thousand-candle-power electric lamp at a distance of about ten feet, and hence abundant for all visual purposes. In fact, as seen from Neptune, the sun would look very much like an electric arc-lamp at a distance of a few yards.

As a source of heat, however, the sun would not amount to much. If the planet's surface were a good absorber and radiator, the solar radiation would only suffice to keep it at a mean temperature of about 51° absolute, or − 222° centigrade, and under these circumstances nitrogen would be a solid and oxygen a solid or a dense liquid (§ 612). The actual temperature of the planet's surface may be considerably raised by the escape of heat from the interior, but nevertheless is probably very low.

None of the other planets of the system could be seen nearly as well from Neptune as from the earth. To eyes like ours Jupiter

solution of the problem is not always unique, as it was in the case of the discovery of Neptune. The mass of the hypothetical planet comes out less than half that of Neptune; but, even on the most unfavorable assumptions regarding its density and albedo, it ought to be of about the twelfth magnitude, and a conspicuous object on long-exposure photographs, so that, if it really exists, it is rather surprising that it has escaped discovery so long.

Gaillot suspects another and larger planet, rather more than twice as far from the sun as Neptune.

One thing, at least, is definitely proved by these investigations. If there were a planet as large as Neptune within twice the distance of Uranus, or one as large as Jupiter within $2\frac{1}{2}$ times the distance of Neptune, it would produce much larger disturbances in the motion of Uranus than have been observed; and hence no unknown planets exist unless they are considerably smaller.

EXERCISES

1. When Jupiter is visible in the evening, do the shadows of his satellites precede or follow the satellites as they cross the planet's disk?

2. On which limb, the eastern or the western, do the satellites appear to enter upon the disk?

3. How would the brightness of sunlight at the distance of Saturn compare with sunlight on the earth?

4. What would be the greatest elongation of the earth from the sun as seen from Jupiter? from Saturn? from Uranus?

5. What would be the apparent angular diameter of the earth when transiting the sun as seen from Jupiter?

6. What is the rate in miles per hour at which a white spot on the equator of Jupiter, showing a rotation period of $9^h\ 50^m$, would pass a spot with a period of $9^h\ 55^m$?

7. Find the diameter, volume, density, and surface gravity of Neptune, taking the apparent diameter of the planet as $2''.3$, the planet's mass as 17 times that of the earth, the solar parallax as $8''.80$, and the mean distance of Neptune from the sun as 30.07.

telescope in 1609, and therefore must have been bright. With the improvement of telescopes and the multiplication of observers

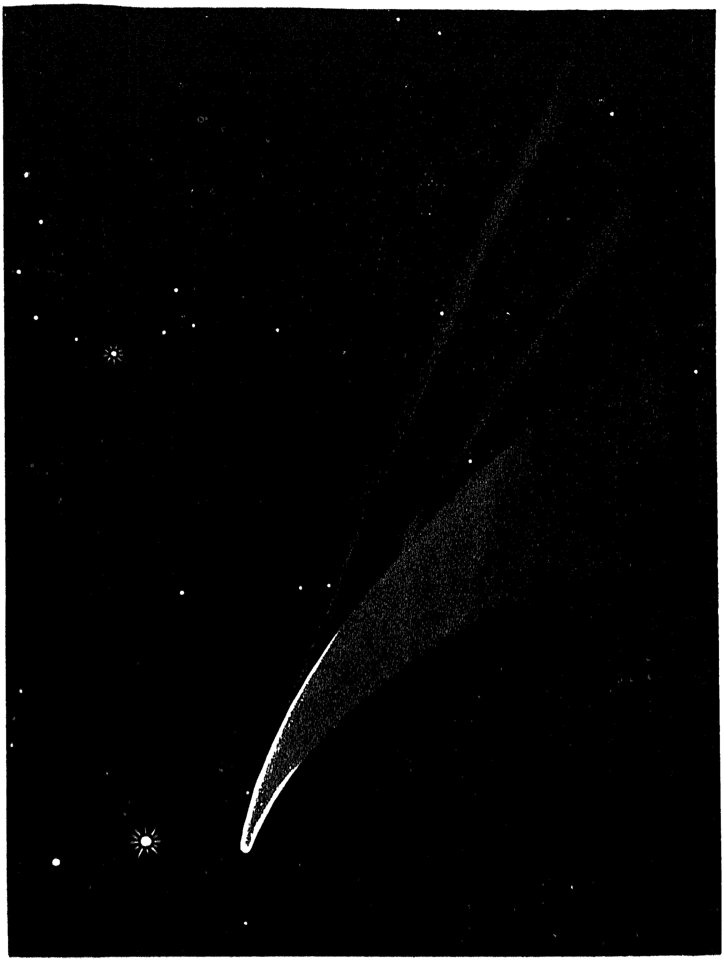


FIG. 163. Donati's Comet

A naked-eye view, October 4, 1858. (From a drawing by Bond)

the rate of discovery has steadily increased. During the last half of the eighteenth century it averaged about one comet per year. Since 1880 the average rate has been a little over five per year, of which 70 per cent were "new" and 30 per cent were returns of

Some observers have found a great number of these bodies, the record being 27, by Pons. Many have been found by amateurs with small telescopes. Occasionally a bright comet is picked up with the naked eye by someone not an astronomer at all, a notable instance being the great comet of January, 1910, which was first seen by three South African railroad workmen.

Recently several have been discovered by photography, — the first by Barnard at the Lick Observatory in 1892, — and in more than one case, upon searching in the position calculated after the orbit became known, images of a comet have been found on photographs taken before its discovery. Halley's comet was photographed in 1909 before it could be observed visually.

481. Duration of Visibility and Brightness. The great comet of 1811 was observed for seventeen months, and that of 1861 for nearly a year. With the more powerful telescopes of the present day, comets can be followed longer. Comet 1889 I was followed for more than two years and a half, up to a distance of more than eight astronomical units from the sun, and the observations of comet 1905 IV, including photographs taken more than a year before its discovery, extend over three years and a half. In most cases, however, comets are lost to sight after a few months; and when one is not discovered until it is receding from the earth or the sun, it is sometimes observable for only a few weeks or days.

On two occasions a comet near the sun has been photographed during a total eclipse, and never seen before or after the few minutes of totality.

As to *brightness*, comets differ widely. Some are barely visible with large telescopes. The majority known are observable with smaller instruments, — which might be expected, since most of the discoverers have worked with relatively small telescopes. About one in five (to judge by the experience of the past thirty years) becomes visible to the naked eye for at least a few days. Fifteen or twenty in a century become so conspicuous that a casual stargazer could hardly fail to notice them, and a very few (perhaps two in a century, on the average) are visible even in broad daylight when near the sun. The last comets so observed were 1843 I, 1882 II, and 1910 I.

the computed parabola by an amount sensible to observation during the time that it was visible. Now most comets are visible only in those very small portions of their orbits which lie near the earth and the sun, and in this portion of the orbit a parabola, an elongated ellipse, and a hyperbola may almost coincide (Fig. 164).

When an "observed arc" is short, it is therefore usually possible to satisfy the observations, within reasonable limits, by means of a variety of orbits, — elliptical, parabolic, or hyperbolic. In this case the parabolic orbit, which is the easiest to compute, is adopted. An orbit is described as elliptical or hyperbolic only when the observations cannot be satisfied by a parabola. Evidently, the longer the observed arc is, the more favorable is the opportunity of finding the real character of the orbit; and this is fully confirmed by experience. Among the best-observed comets elliptic or hyperbolic orbits are found to be the rule, and those indistinguishable from a parabola the exception.

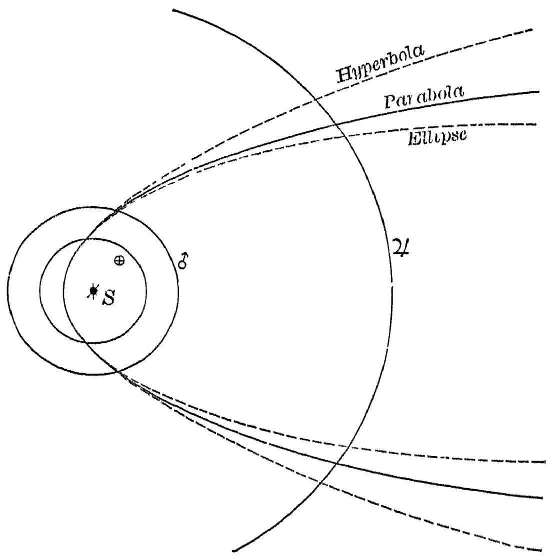


FIG. 164. The Close Coincidence of Different Species of Cometary Orbits within the Earth's Orbit

The earth's orbit is indicated by the sign ⊕

Second, the orbits given by the computers are what are called "osculating" orbits; that is, they are the orbits that each comet would pursue if, at some specified date near the middle of the observations, *all the planets should cease to attract the comet*, leaving it free to move under the influence of the sun's attraction alone.

uncertainty is a week or two; for a period of fifty or a hundred years it is a year or more; and for a thousand-year period it is likely to be a century. It is therefore not surprising that most of the periodic comets observed at a second appearance have been independently discovered, as new objects, and only recognized later as returns of known ones. The recognition often takes some time, for a comet has no individual identity by which it may be recognized merely by looking at it — no striking individual peculiarities like those of the planets Jupiter and Saturn. It is identifiable only by its path.

485. The Short-Period Comets. Of the 54 comets of period less than a century which were known in 1925, 41 have periods between three and nine years in length, and the periods of 36 fall in the much narrower interval between five and seven and one-half years. These short-period comets form a distinct group. Their motions are all direct, and the inclinations of their orbits small, — only three exceeding 30° and the average being 14°. They are all relatively faint objects, few of them being visible to the naked eye, and that only when they come unusually near the earth. A few have at times developed short tails, but most of them have no tails at all.

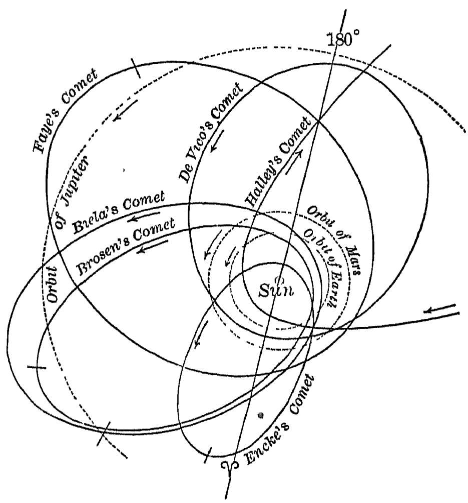


FIG. 165. Orbits of Short-Period Comets whose Aphelia and Nodes are near the Orbit of Jupiter

Fig. 165 shows the orbits of several of these comets (it would cause confusion to insert all of them). It will be seen that in every case (except Halley's comet) the comet's aphelion is not far from Jupiter's orbit, and that one of the nodes (which are marked on the orbits by short cross-lines) is still nearer. It follows that the orbit of each of these comets comes close to that of Jupiter in space, so that if the two bodies pass near the point of

comets of one group to be simultaneously very near one another and near Jupiter also. It is not unlikely that each of these groups has been produced by the disruption of a single comet at some past epoch.

487. Capture of Brooks's Comet. Several instances of the profound alteration of a comet's orbit by an encounter with Jupiter have actually occurred. Brooks's comet (1889 V) was a faint but well-observed telescopic comet, and was seen again in 1896, 1903, 1910, and 1925, its period being 7.10 years. The calculations of Chandler and C L. Poor show that on July 20, 1886, the comet made an exceedingly close approach to Jupiter, — passing inside the orbit of the fifth satellite and only 55,000 miles from the planet's surface. Before this encounter its period was 29 2 years and its perihelion distance 5.5 astronomical units, — almost exactly the same as the aphelion distance afterwards. At its first appearance it was accompanied by four faint companions, which appear to have separated from it at the time of the close approach to Jupiter.

It is clear from the foregoing that most of the short-period comets can hardly be regarded as permanent members of that group, since their orbits may be radically altered by encounters which are almost certain to occur, sooner or later.

488. Acceleration of Encke's Comet. The motions of almost all the comets appear to be just what would be expected of masses moving in free space under the laws of gravitation. But there is one remarkable exception. Encke's comet, a rather faint object, just visible to the naked eye under the most favorable conditions, is noteworthy for its very short period of 3.30 years, — the least so far known. It was seen in 1786, and again in 1795 and 1805, but its periodicity was first recognized by Encke in 1819. Since that date it has been observed at every one of the thirty-one succeeding returns (up to and including 1924). Encke found that after exact allowance was made for the perturbations due to the attraction of the planets (which sometimes alter the period by as much as a week) there remained outstanding a steady shortening of the period, which could not be explained by the attraction of any known body; and this has been fully confirmed by later researches. Between 1819 and 1914 the period has shortened by

shown that (if the resistance to its motion was the same in the past as at the time of discovery) the comet's aphelion was close to Jupiter's orbit about 6000 years ago. It may have been captured by Jupiter at that time, and then saved, by the slow shrinkage of its orbit, from the danger of being sent away into space by another encounter.

489. The comets of longer period differ in several respects from the group just discussed. Many of them have been *conspicuous objects*, far brighter than any short-period comet. Their orbital *inclinations are high*, and the *eccentricities are large*. Several of them are *retrograde*.

Not one of the thirty-six comets whose periods lie between 10 and 1000 years comes nearer than 19,000,000 miles to Jupiter's orbit, and only eight approach it within 50,000,000 miles. The situation with respect to the other major planets is similar.

There appears, therefore, to be little or no evidence that these comets of longer period owe the elliptical character of their orbits to encounters with the planets, unless, indeed, they were captured so long ago that the gradual accumulation of minor perturbations in the interval has shifted their orbits clear away from the original points of encounter. They are probably much more nearly permanent members of our system than are the short-period comets.

490. Halley's Comet. This, in many respects the most famous of all comets, deserves special description.

It was the first periodic comet whose return was predicted. Halley based his prediction upon the fact that he found its orbit in 1682 to be nearly identical with those of the comets of 1607 and 1531, which had been carefully observed by Kepler and Apian; and he also found records of the appearance of bright comets at similar intervals in 1456, 1301, 1145, and 1066. He noticed, of course, that the two intervals between 1531 and 1607, and between 1607 and 1682 were not quite equal, but he had sagacity enough to see that the differences were no greater than might be accounted for by the attractions of Jupiter and Saturn.

Though it was not then possible to compute just what the effect of these attractions would be upon the return of the comet, he saw that Jupiter's action would retard it, and he

could be made. Clairaut, after a most laborious investigation, fixed upon April 13 for the perihelion passage, but remarked that this result might easily be a month out of the way on account of the uncertainty of the masses of the planets (Uranus and Neptune were then unknown). The comet actually came to perihelion on March 13. At this return it was best seen in the southern hemisphere, and had at one time a tail 50° long. At its next return in 1835 it came within two days of the predicted time. It did not appear extremely brilliant but was fairly conspicuous, with a tail about 10° in length.

Its latest return to perihelion occurred on April 20, 1910. Astronomers knew beforehand exactly where to look for the comet, and it was detected photographically by Wolf, at Heidelberg, on September 11, 1909, while still 310,000,000 miles from the sun and a little farther from the earth. It was found later that it appeared on a photograph taken at Helwan, Egypt, on August 24. On its return journey it was followed photographically till July 1, 1911, when it was 520,000,000 miles from the sun. It was lost to sight visually about a month earlier.

On May 19, 1910, a month after the perihelion passage, the comet passed directly between the earth and the sun, transiting the sun's disk (§ 506). Its minimum distance from the earth, 14,300,000 miles, was reached on the following day. During the early part of May it was a magnificent object in the morning sky, growing larger and brighter day by day as it approached the earth, until, a week before the transit, its head was as bright as the brightest stars, and its tail 60° long. After the sixteenth the head was too near the sun to be seen, but the tail continued to be visible in the morning sky, before the head had risen, as a great band of light, almost as broad and bright as the Milky Way, extending clear across the heavens to a distance of more than 120° from the head. For two days after the comet's head had passed to the opposite side of the line joining the earth and sun the tail remained visible in the east, showing that it must have been strongly curved and must have lagged far behind the comet's radius vector (Fig. 169).

It is probable that the earth grazed the edge of the tail, or perhaps passed through it, on May 21, 1910.

50,000,000 miles of the orbit of either planet, and not one of the eight comets supposed to belong to Neptune can ever get within 350,000,000 miles of it if they follow their present orbits. It seems, therefore, very unlikely that any of these eight comets has ever been captured by Neptune. Saturn and Uranus may be responsible for a comet or two, but very few in comparison with Jupiter.

According to the capture theory the great majority of the comets captured by any planet should undergo much less than the maximum possible retardation. If their orbits were originally parabolic, their periods after capture would usually be longer than that of the planet, the very short periods representing exceptional cases. Since Jupiter's family appears actually to consist almost entirely of comets of very short period, it is not unlikely that the material of which they were composed was moving, before its capture, not in parabolic orbits but in elliptical orbits within the solar system.

493. Comet Groups. There have been several instances in which a number of comets, certainly distinct, have followed one another along almost exactly the same path, at intervals varying from a few months to many years. The existence of such groups was first pointed out by Hoek, of Utrecht, in 1865.

The most remarkable group of the sort is composed of the great comets of 1668, 1843, 1880, 1882, and 1887. These are noteworthy for their very small perihelion distances, — 510,000 to 720,000 miles. They passed within 300,000 miles of the sun's surface and *through the corona* with a velocity of about 500 kilometers per second. (Only one other comet, that of 1680, has ever come anything like so near the sun, and its orbit is entirely different from the others.) Their orbits are exceedingly elongated, so that the comets enter the solar system along nearly straight lines, and all from almost exactly the same direction in space, though their orbit planes are inclined a few degrees to one another.

Yet, although the elements of these five comets are almost identical, they cannot possibly all be appearances of the same body. The comets of 1843 and 1882 were very brilliant and were accurately observed. In both cases the orbits are unquestionably elliptical but of long period, that of the first being probably between 400 and 800 years, and that of the second, before it broke

numbers in the first three intervals should be equal. The observed differences probably mean that remote comets are not likely to be discovered. If allowance is made for the fact that comets of smaller perihelion distances may be unfavorably placed and so miss discovery, it is probable that not more than 1/8 of those which come within 5 astronomical units of the sun are picked up. The greatest observed perihelion distances are 4.05 for the comet of 1729, and 4.18 for the comet 1925*a*.

495. The Hyperbolic Comets. There are about twenty comets whose orbits appear to be more or less certainly hyperbolic, and this number is steadily increasing as more comets are accurately observed.

With regard to their inclinations, perihelion distances, and the like, they resemble the majority of parabolic comets. The greatest deviation from a parabola occurs in the case of comet 1886 III, which has an eccentricity of 1.0130; but this comet was observed for only 33 days, and its orbit cannot therefore be determined with the highest degree of precision. According to Strömgren there are only eight cases (up to 1914) in which the deviation from a parabola is so great, in comparison with the observational uncertainty, as to be unquestionably real. These hyperbolic orbits are, however, *osculating* orbits (§ 332), and the computations of Strömgren and Fayet show that in every one of the eight cases the comet's velocity had been increased by the attraction of Jupiter and Saturn while it was approaching the sun, and that the hyperbolic character of the observed orbits was due to this cause alone. In most cases the original orbits at a great distance from the sun were distinctly elliptical, and in the others the assumption that the original orbit was an ellipse of very long period was consistent with the observations, though not demanded by them. There remains, therefore, no conclusive evidence that any comet has ever *approached* the sun along a hyperbolic orbit. The observed hyperbolas have all been produced by planetary perturbations within a very few years before the perihelion passage. Unless, however, the attraction of the planets retards the motions of these comets, while they are retreating from the sun, to almost the same extent that they have previously been accelerated, they will *leave* the

THE COMETS THEMSELVES

497. Physical Characteristics of Comets. The orbits of these bodies are now thoroughly understood, and their motions are calculable with as much accuracy as the nature of the observations of these hazy bodies permits; but we find in their physical constitution and behavior some problems which have not yet received a satisfactory explanation.

FIG. 168. Comet 1910*a*

Photographed January 26, 1910, exposure, ten minutes. The sharp curvature and intricate structure of the outer part of the tail are unusual. The dark objects in the lower part of the picture are pine trees, blurred by the motion of the camera in following the comet. (From photograph by C. O. Lampland, Lowell Observatory)

While comets are evidently subject to the attraction of gravitation, as shown by their orbits, they also exhibit evidence of being acted upon by powerful *repulsive* forces emanating from the sun. While they shine partly by reflected light, they are also certainly *self-luminous*, their light being generated in a way not yet thoroughly explained. They are the *bulkiest* bodies known, except the nebulæ, in some cases thousands of times larger than the sun or stars; but in mass they are "airy nothings," and one of the smaller asteroids probably rivals the largest of them in weight.

498. Photography of Comets. Much of our recent advance in knowledge regarding the physical characteristics of comets is due

away from the sun, although its precise form and position are determined partly by the comet's motion. It is practically certain that the tail consists of extremely rarefied matter, which is thrown off by the comet and then powerfully repelled by the sun.

Thus the tail may make any angle whatever with the direction of the comet's motion, just as the smoke-trail of a slow steamer at sea, consisting of fine sooty particles emitted from the funnels and carried by the wind, stretches always to leeward but may make any angle with the vessel's course.

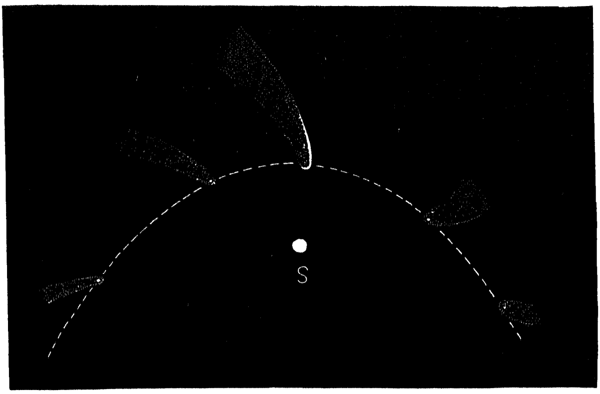


FIG. 169. The Tail of a Comet is directed away from the Sun

There is no sharp distinction between the coma and the part of the tail nearest the comet; one forms the continuation of the other.

(4) The head of a brilliant comet is often veined by *jets* of light, which appear to be continually emitted by the nucleus; and sometimes it throws off a series of concentric *envelopes* like hollow shells, one within the other (Fig. 173). These phenomena, however, are seldom observed in any but bright comets.

500. Dimensions of Comets. The volume of a comet is often enormous. As a general rule the *head*, or coma, is from 30,000 to 150,000 miles in diameter. The average diameter of a large number tabulated by Holetschek is 80,000 miles.

A comet less than 10,000 miles in diameter is very unusual; in fact, such a comet would be almost sure to escape observation. Some are much larger than 150,000 miles. The head of

504. Changes in Size and Form. Certain comets have changed in diameter, as well as in brightness, in an extraordinary fashion.

Holmes's comet (1892 III), when discovered, on November 6 of that year, was visible to the naked eye and about 200,000 miles in diameter, with an outer nebulosity 700,000 miles across, and with a central condensation but no nucleus. A month later it had doubled in diameter; but then it grew so faint and transparent that it could hardly be observed even with large telescopes. In the middle of January it suddenly contracted into a mere hazy star, with a strong nucleus, and with a coma 30,000 miles across (greatly increasing in brightness at the same time), and then gradually expanded to a diameter of 300,000 miles and faded out. At its returns in 1899 and 1906 it was barely visible in the most powerful telescopes. The cause of these remarkable changes is quite unknown.

Biela's comet, which has a period of $6\frac{3}{4}$ years, was observed in 1772, 1806, 1826, and 1832, and presented no unusual features. In 1846, while under observation, it became pear-shaped and divided into two parts. The twin comets traveled along side by side for more than three months at a distance of about 160,000 miles. Each developed a nucleus, and a tail about half a degree in length. When the comet returned in 1852 the twins were both seen, separated by about 1,500,000 miles, and were observed for a month, sometimes one and sometimes the other being the brighter.

Neither of them has ever been seen again, although they must have returned to perihelion ten times, and more than once under favorable conditions for visibility. Their invisibility cannot be accounted for by perturbations, for they did not come near Jupiter till 1889, and not very near it then. They must simply have *stopped shining*, — not partially, like Holmes's comet, but completely.

Taylor's comet (1916 I) also divided into two parts while under observation. The component which was brighter at first faded out some time before the other. This comet, like Biela's, has a short period, — about $6\frac{1}{2}$ years.

505. Masses of Comets. While the volumes of comets are enormous, their masses are apparently insignificant, — in no case comparable with those of the smallest of the planets.

comet 100,000 miles in diameter, and even very near its nucleus, with no perceptible diminution of their luster; and in at least one case this fact has been confirmed by exact photometric measurements.

Still more conclusive is the fact that the great comet of 1882, and Halley's comet in 1910, transited over the face of the sun, and that during the transits both comets were *absolutely invisible.* Even the nucleus was so transparent that no trace of it could be found, in spite of careful searching, — in the latter case by photography and with the spectroheliograph, as well as visually.

As for comets' tails, their density presumably is vastly less than that of the heads, and far below the best vacuum that we can yet produce by any artificial means. It is nearer to an airy nothing than anything else we know of. Schwarzschild, from the observed brightness of the tail of Halley's comet in 1910, has calculated that there could not have been more material in 2000 cubic miles of the tail than in a single cubic inch of ordinary air, and there may have been very much less!

We must bear in mind, however, that the low mean density of the comet does not necessarily imply that the density of its constituent parts is small. A comet may be in the main composed of small heavy bodies, and still have a very low mean density, provided they are widely enough separated. There is much reason, as we shall see later (§ 520) for supposing that this is really the case.

Another point should be referred to. Students often find it hard to conceive how such impalpable "dust clouds" can move in orbits like solid masses and with such enormous velocities; they forget that in a vacuum a feather falls as swiftly as a stone. Interplanetary space is a vacuum, far more perfect than anything we can produce by artificial means, and in it the lightest bodies move as freely and swiftly as the densest, since there is nothing to resist their motion. If the moon, and all the earth, were suddenly annihilated, except a single feather, the feather would keep on and pursue the same orbit, with the unchanged speed of 18½ miles a second.

507. The Brightness of Comets. No other heavenly bodies differ so enormously in brightness as do comets. Some are barely

509. Relative Brightness of Different Comets. From what has just been said it is evident that if we wish to obtain a real measure of the relative brightness of different comets, we must compare their brightness when at the same distances from the sun and the earth. When this is done, it is found that the differences are very great.

The most noteworthy comets of the last four centuries, measured by this standard, were those of 1577, 1744, 1811 I, and 1882 II. It may reasonably be estimated that each of these, at unit distance from the sun and earth, would have appeared

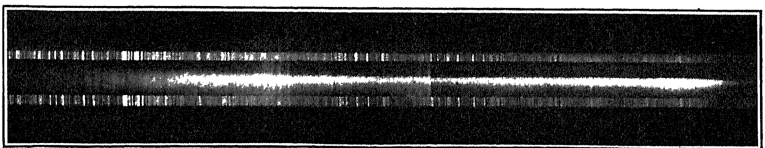


FIG. 170 *A* Spectrum of Halley's Comet, May 6, 1910

Photographed with a one-prism spectrograph attached to the 24-inch Lowell refractor, and on a specially sensitized plate Shows visual hydrocarbon emission bands (wave-lengths (§ 549) λλ4737, 5165, 5635), sodium bright lines (*D*), and also a strong solar spectrum (*C*, *G*, etc.). The relative strength of the solar lines proves that this comet's brightness is due more to reflected sunlight than to emitted light of its own. (From photograph by Lowell Observatory)

about as bright as the brightest stars, such as Arcturus or Vega. Few other comets, at the same distances, would have been one tenth as bright as these.

The greatest comet yet recorded, however, was undoubtedly that of 1729, which was faintly visible to the naked eye, although its perihelion distance was more than four astronomical units. The four comets mentioned above would have appeared, on the average, only about one twentieth as bright at such a distance as the comet of 1729, and it seems certain that if the perihelion distance of this comet had been small, its brilliancy would have surpassed anything on record.

Most of the conspicuous comets owe their brightness to a relatively close approach to the sun or the earth.

510. Origin of the Light of a Comet. Spectroscopic study shows that the light of a comet's head arises partly from *reflected sunlight*, which gives a continuous spectrum, crossed by the familiar Fraunhofer lines (§ 570), and partly from the light emitted by

It is well to remark that while the bright bands in the spectrum prove the presence in the comet of carbon monoxide and nitrogen,

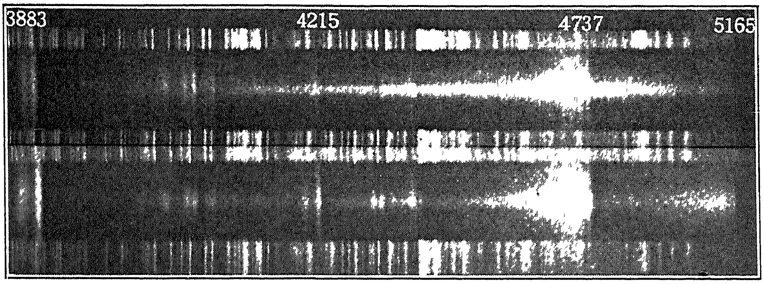


FIG. 171. Spectrum of Mellish's Comet (above), May 9, 1915, and of Zlatinsky's Comet, May 25, 1914

The cyanogen bands λ 3883 and λ 4215 are very different in these comets The carbon band at λ 4737 is nearly the same

and often of hydrocarbons, they do not at all prove that the comet is mainly composed of these substances, or even that such gases constitute a considerable portion of its mass. It is much more likely that solid or liquid particles of various sizes form an overwhelmingly preponderant part of the whole.

511. Development of a Comet as it approaches the Sun. Even a large comet, if first seen at a great distance from the sun, appears as a mere roundish nebulosity, brighter in the middle but usually without a definite nucleus. As it approaches the sun it brightens rapidly, and the nucleus appears. The coma expands, and the tail begins to form, often at the start as a narrow streamer (Fig. 172). For great comets the distance from the sun at which the tail begins to form averages about 200,000,000 miles. As the activity increases, the nucleus, on its

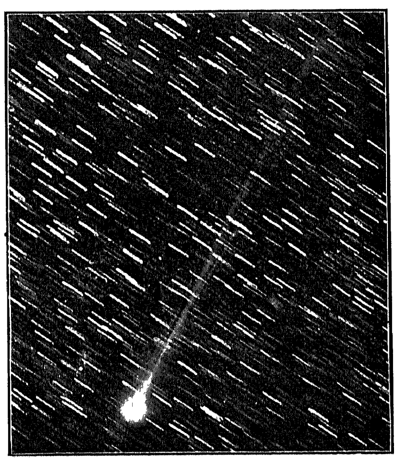


FIG. 172. Gale's Comet, May 5, 1894

In Morehouse's comet of 1908, however, the envelopes lasted but a few hours, and contracted as they grew older; but in this case also several of them were visible at the same time.

The envelopes appear to be formed by material projected from the nucleus on the sunward side, and repelled by the sun in the manner illustrated in Fig. 174 (in which, however, the downward curvature of the paths of the outer particles is greatly exaggerated).

It seems clear that such an envelope, like the jet of a fountain, must be continually composed of new material, though retaining its form unaltered. The only difficulty about this "fountain theory" of the formation of the envelopes is that in certain cases, notably in Morehouse's comet (as Eddington has shown), very high initial velocities and very great repulsive forces are demanded, to account for the rapid formation and changes of the envelopes; but no other theory thus far suggested appears to fit the facts nearly so well.

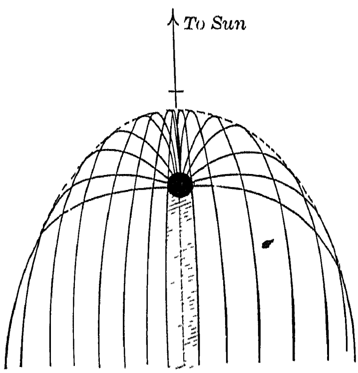


FIG. 174. Formation of a Comet's Tail by Matter expelled from the Head

513. Formation of the Tail. It may be supposed that the material ejected from the nucleus and repelled by the sun sweeps backward to form the tail, along which it streams, still luminous, until it becomes so widely diffused and so rarefied that it is no longer visible.

Comets' tails, therefore, are simply *streams of repelled particles*, each moving in its own hyperbolic orbit around the sun, the separate particles exerting no sensible influence upon one another, and (except perhaps at the very beginning of their course) being quite emancipated from the control of the comet's head.

This view of their nature accounts for the observed shapes and changes of the tails of different comets, and has been conclusively confirmed by direct observation since photography has made it possible to study details visible with difficulty, if at all, to the eye.

In some cases even greater repulsive forces appear to have existed, notably in Morehouse's comet, where detached masses in the tail themselves emitted subsidiary tails evidently composed of matter driven away from them even more rapidly than they themselves were moving.

The low-pressure spectrum of carbon monoxide has been found to be especially characteristic of the narrow, straight streamers, while the more featureless part of the tail, in Halley's comet at least, gave a nearly continuous spectrum.

515. Direct Evidence of Motion in the Tail. The most conclusive evidence that the tails of comets are composed of matter streaming away from the head is given by photography. In about half the brighter comets of the past twenty-five years, luminous knots, or condensations, have at times been observed in the tail. Whenever such an object has been photographed at intervals of even a few hours, it was found to be rapidly receding from the comet's head; and when the observations extended over a longer interval, they showed that its rate of recession was steadily increasing.

Thus, a detached mass in the tail of Halley's comet photographed by Curtis was receding from the head at an average rate of 70 kilometers per second between June 6 and 7, 1910, and at the rate of 91 kilometers per second on the following day.

Velocities of the same order of magnitude have been observed in other comets. They indicate the existence of repulsive forces emanating from the sun (since the acceleration is almost entirely independent of the distance from the comet's head) and of magnitude varying in different cases between 30 and 100 times the sun's gravitational attraction.

In some cases, as in Morehouse's comet, the changes are very rapid (Fig. 176). Comparison of the motions of details at different distances from the head shows that the material is ejected from the nucleus with a relatively low velocity, and develops its high speed later under the action of the sun's repulsion. The end of a tail, many millions of miles long, would be reached by the outgoing material within a week.

From measures of the brightness of the tail of Halley's comet, Schwarzschild and Kron have shown that the gradual fading out

Though the forces which cause the motion of the luminous matter along the tail appear now to be well understood, those which occasion its ejection from the nucleus are still obscure. The initial velocity is often several kilometers per second, — too great to be ascribed to the mere expansion of liberated gases into a vacuum, — and the appearance of the short-lived envelopes in Morehouse's comet suggests the occurrence of almost explosive emissions of gas, with velocities at first very high but dying down in the course of a few hours. The energy demanded by the emission may either be derived from the incident solar radiation or be latent in the material of the head and liberated in some way by sunlight.

517. Anomalous Phenomena. A number of comets have at times possessed anomalous tails (usually in addition to the normal tail, but sometimes substituted for it), sometimes directed straight toward the sun and sometimes nearly at right angles to that direction. Bredichin has shown that tails of the latter sort may be accounted for by changes in the velocity of projection of the particles from the nucleus, while those directed toward the sun may be composed of relatively large particles, emitted at low velocity, for which the light pressure is small in comparison with the gravitational attraction.

The great comet of 1882 also carried with it for a time a faintly luminous sheath, which seemed to envelop the comet itself and that portion of the tail near the head, projecting 2° or 3° toward the sun. For some days, moreover, it was accompanied by little clouds of cometary matter, which left the main comet, like smoke puffs from a bursting bomb, and traveled off at an angle until they faded away It is natural to associate these unusual phenomena with the profound disturbances which the outer parts of the comet must have suffered during its precipitate flight around the sun at perihelion, when it went through 180° of orbital longitude in less than four hours.

518. Dissipation of the Cometary Material. *There is not the slightest reason to suppose that the matter driven off to form the tail is ever recovered by the comet.* The minute particles must fly off into space at very high velocities. A very small fraction of them may ultimately be picked up by dark bodies in space (though not by bright stars, whose light-pressure would repel them), but the overwhelming majority must wander alone in interstellar space indefinitely.

It follows that such comets as have tails lose a portion of their substance every time they visit the neighborhood of the sun. It

which grows longer and brighter as the comet approaches perihelion. The emissions from the nucleus sometimes take the form of jets, or streams, and sometimes the outflow is more regular, resulting in the formation of envelopes.

As the comet withdraws from the sun, its history is reversed, except that the development of the activity lags a little behind the excitation which produced it, so that the maximum brightness is reached a little after the perihelion passage and the comet is brighter, at the same distance from the sun, after perihelion than before. Finally, as it recedes into the asteroid zone, the internal activity ceases; the remnants of gases and fine dust are blown away into space by radiation pressure; the coarser dust perhaps settles down or agglomerates to some degree; and the comet is left nearly in its original state during the long interval before the next perihelion passage.

520. Photometric Estimate of a Comet's Mass and Density. On this theory of a comet's constitution it sometimes becomes possible to make an estimate of the order of magnitude of its mass from measures of its brightness.

Take for example, Halley's comet. In September, 1909, it was still apparently inactive, and probably shone by reflected sunlight. Its measured light was about equal to that which would have been reflected from a single body, in the same position, of the albedo of the moon and 40 kilometers in diameter; but the comet itself was 22,000 kilometers in diameter and presented 300,000 times as large an area.

It follows that if we could have magnified the comet sufficiently to see the separate particles of which it was composed, we should have found them to be scattered very sparsely over the dark background of the sky, covering probably not more than 1/300,000 of the whole area within the comet's apparent boundary. It is no wonder that the comet was transparent.

We can do little more than guess what the size of the particles may be, but it is not at all probable that they average less than a centimeter in diameter, for if they were smaller the radiation pressure upon them would be a sensible, though small, fraction of the sun's attraction, and would produce perceptible modifications in the comet's orbital motion.

It may readily be computed that a small, rapidly moving body which approaches the sun within one astronomical unit stands about one chance in 400,000,000 of hitting the earth. As about five comets come within this distance every year, the nucleus of a comet should hit the earth, on the average, once in about 80,000,000 years. Collisions with the outer parts of the head should be many times more frequent.

As to the consequences of such a collision it is impossible to speak positively for want of sure knowledge of the constitution of the comet. If the theory which has been presented is true, everything depends on the size of the separate particles which form the main portion of the comet's mass. If they weigh tons, the bombardment experienced by the earth when struck by the comet would be a serious matter, although it would probably fall very far short of producing a wholesale destruction of terrestrial life. If, as seems more likely in the case of the outer portions, they are for the most part as small as pinheads, the result would be simply a splendid shower of shooting stars.

A danger of a different sort has been suggested, — that if a comet were to hit the earth, our atmosphere would be poisoned by mixture with the gaseous components of the comet. Here again the probability is that on account of the low density of the cometary matter no sufficient amount would remain in the air to do any mischief at the earth's surface. Moreover, combination with the oxygen of our atmosphere would render quite harmless any of the gases whose presence has been detected in comets.

As for encounters with comets' tails, they are probably of frequent occurrence. It is certain that the earth passed through the tail of the great comet of 1861, and it is probable that it at least grazed that of Halley's comet in 1910. In neither case was any perceptible effect produced.

522. Effect of the Fall of a Comet into the Sun. As to this it may be stated that except in the case of Encke's comet there is no evidence of any action going on that might cause a periodic comet to strike the sun's surface; it is doubtless possible, however, that a comet may sometimes enter the system from a distance, so accurately aimed as to hit the sun.

is generally followed by a luminous train, which sometimes remains visible for many minutes after the meteor itself has disappeared. The motion is sometimes irregular, and here and there along its path the fire-ball throws off sparks and fragments. Sometimes it vanishes by simply fading out in the distance, sometimes by bursting like a rocket. By day the luminous appearances are mainly wanting, though sometimes a white cloud is seen, and even the train may be visible.

524. Nature of Meteors. The large apparent velocity with which meteors move indicates at once that they must be relatively very near us. This is fully confirmed when the same meteor has been observed at different stations and its parallax found. It thus is shown that visible meteors are within a hundred miles or so of the earth's surface.

They are small bodies which approach the earth from interplanetary space at high velocity, and shine only when they enter the upper atmosphere and become heated by friction in flying through the air. Most of them are soon consumed, burning completely away. Those which get through to the earth's surface are called *meteorites*. A number have been recovered which were actually seen to fall, and others have been identified by their resemblance to these.

525. Number. Meteors come from all parts of the sky, as well as in showers from small areas. The number of *sporadic* shooting stars is enormous. A single observer averages from four to eight an hour; if accustomed to such observation and well situated, he may see twice as many on a moonless night.

Estimated on the basis of individual counts, the total number which enter our atmosphere in twenty-four hours and are bright enough to be visible to the naked eye must be several millions, and in addition there is probably a still larger number so small as to be observable only with the telescope.

The average hourly number after midnight is about double the hourly number in the evening. In the morning we are on the front of the earth in respect to its orbital motion. We see at that time meteors which the earth overtakes, as well as those coming to meet it. The apparent velocity of the morning meteors averages high for the same reason. In the evening we see only

into altitude and azimuth. The time of flight is hard to estimate with any approach to accuracy. Some observers begin to repeat rapidly some familiar verse of doggerel when the meteor is first seen, reiterating it until the meteor disappears.

Sometimes trails of bright meteors are caught accidentally on photographic plates, and special apparatus has been devised and successfully used in photographing meteors. Such observations are of course much more accurate than visual ones.

527. Paths of Meteors. Visual observations are necessarily of low accuracy; but when several competent observers have seen the same meteor, it is possible to find the position of its path and the heights of appearance and disappearance within a few miles.

Such data show a dependence of the heights of appearance and disappearance, and of the length of flight, upon the apparent size of the body and its velocity. The smaller ones, or *shooting stars*, appear at an average height of about 70 miles and disappear about 50 miles from the earth's surface, after an average flight of 35 miles. For certain groups, of high velocity, the altitudes of appearance and disappearance are greater. The corresponding heights for *fire-balls* are 85 and 30 miles, and the length of path averages something like 200 miles. Some of them travel much farther.

Since large masses must suffer relatively less resistance from the air and be completely consumed less rapidly than small masses, it is reasonable to conclude that fire-balls and meteorites are much more massive than shooting stars.

The flight of an average shooting star occupies so brief a time that direct velocity determinations are difficult and inaccurate. The apparent paths of fire-balls and meteorites are usually much longer and the observations better. Velocities as low as 15 km./sec., and as high as 75 km./sec., have been observed. The velocities with which meteors enter the atmosphere must be rapidly reduced as they penetrate deeper into the denser layers. No appreciable slowing up toward the end of flight of shooting stars is observed (largely owing to the briefness of their visibility), but it is often distinctly apparent in the case of meteorites. By the time these come within a few miles or so of the earth's surface the air resistance is so great that they are

529. Masses of Meteors. The luminous energy radiated by a meteor may be calculated from observations of its apparent brightness at a known distance and the duration of its visibility. If we knew what fraction of the kinetic energy of the meteor was transformed into luminous energy, we could find the kinetic energy and, knowing the velocity, the mass of the meteor. Only a rough guess at this fraction can be made; but, unless it is very much smaller than for other light sources of the same color, the mass of a typical shooting star is only a few milligrams.

In the case of large meteorites, which weigh many kilograms when found, in spite of the losses which they sustain in the atmosphere, sufficient energy is available to make the light radiated millions of times greater than from the average shooting star. This is entirely consistent with reports that large fire-balls have appeared much brighter than the full moon.

But while meteors are, on the average, very small, they are almost countless. The total annual amount of meteoric matter which falls on the earth, measured in tons, must be large; but as an addition to the earth's mass it is trivial, and its effects on the revolution and the rotation of the earth are too small to be measurable.

530. Meteorites. The number of instances in which meteorites have actually been seen to fall, and have been recovered, has averaged about four per year since 1850. But for one that is found, even of the fire-balls whose flight has been observed, a dozen are missed; and if we include all that presumably were not seen, or that fell unobserved into the ocean or in regions from which no report could come, the sum total must be very great. Their number is small, however, compared with that of shooting stars.

The mass that falls is sometimes a single piece, but more usually there are many pieces, sometimes to be counted by thousands. At the Pultusk fall, in 1869, the number of meteorites was estimated to exceed 100,000, most of them very small. The largest single mass, so far as known, is one of three brought to the American Museum of Natural History, in New York, from Melville Bay, Greenland, by Admiral Peary. Its weight is $36\frac{1}{2}$ tons, and its approximate dimensions are $10.9 \times 6.8 \times 5.2$ feet.

(up to 1915) and were recovered for study were composed of iron. About half the specimens in museums, however, are iron meteorites, since these, even though they may have fallen centuries ago, are recognizable at once as remarkable objects, while stony meteorites, except to the trained observer, look like any other stones.

About thirty of the chemical elements, including argon and helium, have been found in meteorites. Some of the elements are combined to form minerals not found in terrestrial rocks, but no otherwise unknown chemical elements have been found in meteorites. Of those present, iron, oxygen, nickel, silicon, and magnesium are by far the most abundant, in the order given; then sulphur, calcium, cobalt, aluminium, and sodium.

When heated, meteorites give out gases, including hydrogen, nitrogen, carbon monoxide, and hydrocarbons. Some meteoric minerals, notably a phosphide of iron and nickel, could not have been formed in presence of free oxygen. The iron meteorites usually show a crystalline structure, and the stone meteorites a structure of rounded crystalline grains, both of which are distinctive of these bodies. These structures indicate that the material must once have cooled from a melted condition, — rapidly for most stone meteorites and slowly for most of the irons.

532. Groups of Meteorites. Meteorites are small compared with their apparent size when rushing through the air. Then the surrounding luminous shells of intensely heated vapor and air contribute to the apparent size. But this is perhaps not all. Sometimes the pieces picked up near the same locality show fresh, unfused fracture surfaces, evidently resulting from the bursting of a single body. More often, and especially in cases where a great many stones fall, they all have the same characteristic surface resulting from fusion, showing that they entered the atmosphere as a group of separate bodies. In such a case the detonation heard at the end of the visible flight is not the result of explosive bursting of the meteorite, but rather of the sudden equalization of air pressure.

The hypothesis, first advanced by Haidinger and Galle, that a meteor is usually not a single body, finds further confirmation in the observation of groups of meteors, particularly in the

iron and nickel, a combination that is rarely found elsewhere than in meteoric material.

The evidence appears very strong that this crater has been produced within modern times, geologically speaking, by the impact of a great mass of meteoric material, perhaps a swarm and not a single body, which, too heavy to be stopped by air resistance, carried energy enough to excavate a hole more than half a mile across and a thousand feet deep (counting to the undisturbed rocks below the crushed material). Trees growing on the rims are as much as seven hundred years old, and it is probable that the impact occurred a few thousand years ago.

FIG 181 Meteor Crater, Arizona
From photograph by D. M Barringer

The swarm which produced this crater must have been so compact as to have been invisible at planetary distances. It is noteworthy that nearly every one of the large iron meteorites found on the earth's surface lies on the side of our planet which, at the time of the great impact, faced in the direction from which this swarm came. They *may* all have belonged to one large swarm.

534. Meteoric Showers; Radiants. There are occasions when the shooting stars, instead of appearing here and there in the sky at intervals of several minutes, appear in *showers* of thousands. Members of such showers do not move at random, but all their paths diverge, or radiate, from a single point in the sky, known as the *radiant*; that is, their paths produced backward all pass through or near that point, though they do not usually start there (Fig. 182).

orbital motion of the earth; but if the velocity of the meteors is known, their motion relative to the sun can be deduced.

Owing not only to errors of observation but also to the effect of the earth's attraction in changing the paths of the meteors, and probably also to lack of exact parallelism of the paths, the observed radiant is usually a small area of the sky rather than a point.

Probably the most remarkable of all the meteoric showers that have been recorded was that of the Leonids, on November 12, 1833. The number seen at some stations was estimated as high as 200,000 an hour for five or six hours. "The sky was as full of them as it ever is of snowflakes in a storm," and, as an old lady described it, looked "like a gigantic umbrella."

535. Dates of Meteoric Showers. Meteoric showers from the same radiant habitually recur on or about the same day of the year. Some of the radiants, notably that of the Perseids, are about equally active every year. From other radiants come a big shower one year and a rapidly diminishing number in the immediately preceding and following years, but always on the same date. This proves that the meteoric swarms pursue regular orbits around the sun, and that the annual shower occurs at the point where the earth's orbit cuts the path of the particular swarm. The earth reaches this point at the same date every year (Fig. 183).

In some cases the meteors are distributed in a nearly uniform ring along the whole orbit; for such, as in the case of the Perseids, the shower recurs every year with about the same vigor. On the other hand, the flock may have a distinct concentration with only a few stragglers scattered along the orbit; then a notable shower will occur only in the year when the earth meets this aggregation at the orbit crossing. This is the case both with the Leonids and with the Andromedes, though the latter are already getting widely scattered.

Olivier lists, up to 1920, over 1200 radiants, and expresses confidence in the reality of at least half of these. The most conspicuous of them are the Draconids, January 2; the Lyrids, April 20; the Aquariids I, May 6; the Aquariids II, July 28; the Perseids, August 12; the Orionids, October 20; the Leonids, November 14; the Andromedes, November 24; the Geminids, December 10.

predicted and was observed in Europe; and it was followed in 1867 by another, which was visible in America, the meteoric swarm being extended in so long a procession as to require more than two years to cross the earth's orbit. Neither of these showers, however, was equal to the shower of 1833. The researches of H. A. Newton, supplemented by those of J. C. Adams, the discoverer of Neptune, showed that the swarm moves in a long ellipse with a thirty-three-year period.

A return of the shower was expected in 1899 or 1900, but failed to appear, although on November 14–15, 1898, a considerable number of meteors were seen, and in the early morning of November 14–15, 1901, a well-marked shower occurred, visible over the whole extent of the United States, but best seen west of the Mississippi, and especially on the Pacific coast. At a number of stations several hundred Leonids were observed by the eye or by photography, and the total number that fell must be estimated by tens of thousands. The display, however, seems nowhere to have rivaled the showers of 1866–1867, and these were not to be compared with that of 1833. Very few meteors were seen in 1902, but in 1903 a large number were observed in Greece and in England.

The calculations of Downing and Stoney show that the failure of the Leonids to appear in 1900 was probably due to perturbations of the meteors by the action of Jupiter, Saturn, and Uranus, causing the main condensation to pass at a distance of nearly 2,000,000 miles from the orbit of the earth. The dates of some other showers have shown a gradual change, which is doubtless due to perturbations of the longitude of the node.

537. Identification of Meteoric Orbits with Cometary Orbits. The researches of H. A. Newton and J. C. Adams had awakened lively interest in the subject, and Schiaparelli, a few weeks after the Leonid shower, published a paper on the Perseids, or August meteors, in which he brought out the remarkable fact that they are moving in the same orbit as that of the bright comet of 1862, known as Tuttle's comet. Shortly after this Leverrier published his orbit of the Leonid meteors, derived from the observed position of the radiant in connection with the periodic time assigned by Adams. Almost simultaneously, but without any idea of a

tion of their velocities relative to the sun. The best way of getting at this is by studying the daily variation in the number observed. The greater the average velocity of the meteors, the less, evidently, will be the difference between the number that the earth overtakes and the number that overtake the earth. The weight of the evidence, and especially that recently presented by Hoffmeister, favors an *average* velocity which is strongly hyperbolic.

Meteorite falls and great fire-balls are more often observed tween noon and midnight than in the morning hours, and more often in the spring than in the autumn. This is exactly opposite to the behavior of shooting stars, and indicates that most meteorites overtake the earth and are moving in direct orbits about the sun. Bodies moving in retrograde orbits would have a much higher relative velocity and would be much more likely to be consumed in the earth's upper atmosphere. This may suffice to explain the observed facts.

Direct observations of velocity, which are notoriously hard to make with accuracy, indicate decidedly hyperbolic paths for most fire-balls and meteorites. On the other hand, fire-balls have often been observed during meteoric showers, coming from the same radiant. What proportion of shooting stars and meteorites belong to the solar system, and what proportion are visitors from interstellar space, can only be decisively settled by further study, and in particular by some more accurate method of measuring velocity, if such can be devised.

THE ORIGIN OF THE SOLAR SYSTEM

539. Regularities in the Solar System. The solar system is clearly no accidental aggregation of bodies. The remotest planet is hardly more than one ten-thousandth part as far away as the nearest known star, and all the planets share the rapid motion of the sun through interstellar space (§ 740). It is obvious, therefore, that the sun, the planets, and their satellites — together, probably, with the comets — must have had a common origin. Moreover, the planetary system presents numerous regularities of arrangement, for which the mind demands an explanation,

This happened several times, leaving the shrinking nebula surrounded by rings, the smaller of which revolved more rapidly. Finally, the material of each ring somehow collected into a single body, forming a planet; and the central mass condensed to form the sun.

This theory accounts for many of the facts and can be modified to explain more, but it meets with two fatal difficulties. First, it can be proved that an extended tenuous ring would not condense into a single body, but into many bodies, like the asteroids or the rings of Saturn. Second, almost all the angular momentum of the solar system — 98 per cent of the total — is at present associated with the orbital motions of the major planets. The sun's rotation provides almost all the rest, the four terrestrial planets contributing less than 0 1 per cent of the whole. The total angular momentum cannot be altered by any internal changes within the system, and no process has ever been imagined by which 98 per cent of it could have been segregated in less than 1/700 of the total mass. Furthermore, it has been proved that if the outer parts of the nebula had had so much angular momentum, they could not have condensed at all — even into asteroids.

It appears, indeed, that no orderly process of evolution under the action of internal forces could have produced the existing distribution of angular momentum; and it follows that the angular momentum of the planets must have been *put into the system from outside*. Here, therefore, it seems necessary, for once, to abandon the "uniformitarian" hypothesis of gradual evolution and to adopt a "catastrophic" hypothesis of sudden change.

541. The Hypothesis of Dynamic Encounter. An alternative theory, which appears at present to be much more satisfactory than the "nebular hypothesis," was proposed about twenty years ago by Chamberlin and Moulton, of Chicago. According to this theory the sun was once an isolated star, without planets. At some remote epoch another star, in its motion through space, happened to pass very near the sun. The two bodies swung about one another in hyperbolic orbits, and separated again. Their minimum distance was so small that the tide-raising forces due to the star's attraction of the sun — aided by the expansive force of the solar gases — became great enough to counteract the sun's attraction at some points, so that great quantities of matter were ejected from the sun.

The initial motion of the ejected masses was straight away from the sun, and in the absence of disturbing forces they would all have fallen back again into it; but (as Moulton has shown) the disturbing force due to the star's attraction would, in general, tend to impart to these masses a lateral motion, and

name), and the present planets have grown from much smaller original nuclei by gradual accretion of these small bodies. The newer, or "tidal," form of the theory concludes that the planets must have possessed nearly their present masses from the beginning and that the diffuse material was largely gaseous. The difference is thus rather one of degree than of kind, and the two forms of the theory of dynamic encounter may well be discussed together.

543. Formation of the Planets. The ejected material must originally have been gaseous and exceedingly hot, and would have liquefied and solidified only after it had cooled by expansion and radiation. During this process the lighter and more volatile constituents would escape from all but the larger masses — just as the moon's atmosphere has escaped from the moon. The difference in density between the terrestrial and major planets may be due to differences in composition which have arisen in this way.

Chamberlin believes that the planets, except perhaps the largest, were solid throughout practically from the start. Jeffreys concludes that they must have passed through an intermediate liquid phase, but that the earth, for example, must have solidified within about 15,000 years of its birth. After this the surface began to cool, an ocean formed upon it, and geological history commenced.

544. Eccentricity of the Orbits. The dispersed material which did not condense into the primeval planets plays an important rôle in both forms of the theory. The particles, whether meteorites or molecules, must have been circulating about the sun in the same direction as the planets. Such a revolving swarm, or atmosphere, would have little influence upon the forward orbital motion of a planet (which must have been moving at about the same rate as the swarm in its neighborhood) but would in the long run slow down, as if by frictional resistance, the outward and inward motions arising from the eccentricity of the planet's orbit. The planetary orbits, which were probably at first highly eccentric, would thus become more nearly circular.

The dispersed matter would gradually tend to disappear. The separate planetesimals would be picked up by the planets or

them more nearly circular and probably diminishing their size; but how they became so very nearly circular and so close to the planets' equatorial planes is not yet clear. The outer, retrograde satellites of Jupiter and Saturn may once have been asteroids, and may have been captured with the aid of the "resisting medium," but this is uncertain.

547. Origin of the Moon. The moon presents a special problem, being far larger, in proportion to its primary, than any other satellite, or than any planet compared with the sun. If the earth and moon ever formed one mass, it must have rotated in about four hours and have been greatly flattened at the poles. This rapid rotation would not, by itself, have been sufficient to cause the mass to break up, but if, as Darwin suggested, the natural period of free vibration of the mass was, at some stage in its contraction, the same as that of the tides produced by the sun, these tides would gradually rise to enormous heights, distorting the body and ultimately causing it to divide into two parts. It is not improbable (as Jeffreys has shown) that this may have happened, but the alternative hypothesis that the earth and moon were produced together at the time of the great catastrophe cannot be disproved.

There is a good deal in favor of the theory that some, if not all, of the lunar craters have been produced by the impact of planetesimal bodies of moderate size, relatively late in the moon's career, — though long ago, even as geological time is measured. Similar craters, if produced on the earth, would have been gradually destroyed by denudation, and this may suffice to explain their absence.

548. Present State of the Problem. The cosmogonic hypotheses which have here been summarily sketched are far from resting upon the secure bases of exact calculation which support most of the conclusions of astronomy. The mathematical difficulties of a detailed or precise treatment of problems like these much exceed those of the relatively simple "problem of three bodies" (§ 321), and only rough, qualitative conclusions are possible.

The hypothesis that the planetary system owes its origin to an encounter between the sun and a passing star appears to be

4. The perihelion distance of the great comet of 1882 was 0.00775 astronomical unit. What was the velocity V at perihelion of the component which had a period of 769 years?

Ans. For this component, $a = (769)^{\frac{2}{3}} = 83.9$ astronomical units. By equation (7), p 271, and § 315,

$$V^2 = (29.76)^2 \times (2/0.00775 - 1/83.9) = (29.76)^2 \times (258 - 0.012).$$

Hence $$V = 29\ 76\sqrt{257\ 99} = 477 \text{ km./sec.}$$

5. How much greater was the velocity at perihelion for the component with a period of 875 years if the perihelion distance was exactly the same?

Ans For this component, $a = (875)^{\frac{2}{3}} = 91\ 4$. If V_2 is the velocity of this component and V_1 that of the other, then

$$V_1^2 = (29.76)^2 \times (2/r - 1/83.9),$$
$$V_2^2 = (29\ 76)^2 \times (2/r - 1/91.4),$$

whence $V_2^2 - V_1^2 = (29.76)^2 \times (1/83\ 9 - 1/91\ 4) = 886/83.9 - 886/91.4$
$= 10\ 57 - 9\ 71 = 0.86,$

or $$(V_1 + V_2)(V_1 - V_2) = 0\ 86.$$

But $V_1 + V_2 = 954$; $V_2 - V_1 = 0.0009$ km./sec , or 90 centimeters per second.

6. Will a given comet (say Encke's) have precisely the same orbit on successive returns?

7. Why can we not infer with certainty that two comets which have orbits practically identical are themselves identical?

8. Can we, from spectroscopic observations of a comet, infer the relative proportions of the luminous and non-luminous substances present in the comet?

9. Is it probable that a comet can continue permanently in the solar system as a comet? If not, why not, and what will become of it?

10. If a compact swarm of meteors were now to enter the solar system and be deflected by the attraction of some planet into an elliptical orbit around the sun, would the swarm continue to be compact? If not, what would be the ultimate distribution of the meteors?

11. Assuming that the earth encounters 20,000,000 meteors every 24 hours, what is the average number in a cubic space of 1,000,000,000 cubic miles (that is, a cube 1000 miles on each edge)? *Ans.* About 250.

12. If space were occupied by meteors uniformly distributed 100 miles apart on three sets of lines perpendicular to each other, how many would be encountered by the earth in a day? *Ans.* 78,700,000.

NOTE. In this cubical arrangement the *average* distance between the meteors much exceeds 100 miles. If they were packed as closely as possible, consistently with the condition that the distance between two neighbors *should nowhere be less than 100 miles*, the number would be increased by nearly 40 per cent.

APPENDIX

TABLE II. DIMENSIONS OF THE TERRESTRIAL SPHEROID

(Hayford's Spheroid of 1909)

Equatorial radius $a = 6378\,388$ km. $= 3963.34$ miles.
Polar radius $b = 6356\,909$ km. $= 3949.99$ miles.
Mean semidiameter, $\frac{1}{3}(2a + b), = 6.37123 \times 10^8$ cm. $= 6371.23$ km $= 3958.89$ miles.
Oblateness $\dfrac{a-b}{a} = \dfrac{1}{297.0}\cdot$ (See section 138.)
1° of latitude, ϕ, (in statute miles) $= 69.0569 - 0.3494 \cos 2\phi + 0.0007 \cos 4\phi$.
1° of longitude (in statute miles) $= 69.2316 \cos \phi - 0.0584 \cos 3\phi + 0.0001 \cos 5\phi$.

TABLE III. ASTRONOMICAL CONSTANTS

Length of the day.
Sidereal $= 23^h\ 56^m\ 4^s.091$ of mean solar time.
Mean solar $= 24^h\ 3^m\ 56^s.555$ of sidereal time.
(For conversion of time-intervals see section 44.)
Length of the year (in mean solar units), 1900, Newcomb
Tropical $= 365^d.24219879 = 365^d\ 5^h\ 48^m\ 45^s.98$
Sidereal $= 365^d.25636042 = 365^d\ 6^h\ 9^m\ 9^s.54$
Anomalistic $= 365^d.25964134 = 365^d\ 6^h\ 13^m\ 53^s.01$
The length of the sidereal year is 31,558,149.5 sec $= 3.1558 \times 10^7$ sec.
Length of the month (in mean solar units), according to Brown:
Synodical $= 29^d.530588 = 29^d\ 12^h\ 44^m\ 2^s.8$.
Sidereal $= 27^d.321661 = 27^d\ 7^h\ 43^m\ 11^s.5$.
Nodical $= 27^d.212220 = 27^d\ 5^h\ 5^m\ 35^s.8$.
Obliquity of the ecliptic $= 23°\ 27'\ 8''.26 - 0''.4684\ (t - 1900)$
General precession $= 50''.2564 + 0.000222\ (t - 1900)$ } Newcomb.
Constant of nutation $= 9''.21$
Constant of aberration $= 20''.47$
Solar parallax $= 8''.80$ } Adopted for ephemeris purposes, Paris Conference (1911)
Velocity of light $= 299{,}820$ km./sec. $= 186{,}300$ mi./sec. (Michelson, 1924.)
Constant of gravitation $G = 6.673 \times 10^{-8}$ c.g.s. units
Acceleration of gravity g (in meters) $= 9.8060 - 0.0260 \cos 2\phi - \dfrac{2h}{R}g$ (Helmert),
in which h is the elevation above sea-level in meters, and $\log R = 6.80416$.
Earth's mass $= (5.974 \pm 0.005) \times 10^{27}$ grams.
Sun's mass $= 1.983 \times 10^{33}$ grams.
Sun's mean radius $= 6.953 \times 10^{10}$ cm.
1 astronomical unit (A.U.) $= 1.4945 \times 10^8$ km $= 9.2870 \times 10^7$ miles.
1 light-year $= 6.3310 \times 10^4$ A.U. $= 9.463 \times 10^{12}$ km. $= 5.88 \times 10^{12}$ miles. (§ 714.)
1 parsec $= 3.258$ light-years $= 2.06265 \times 10^5$ A.U. $= 3.084 \times 10^3$ km.
$= 1.92 \times 10^{13}$ miles. (§ 714, Vol. II.)

OF THE SOLAR SYSTEM

January $0^d.0$ G.M T)

Mean Orbital Velocity (Km /Sec)	Eccentricity	Inclination to Ecliptic	Longitude of Ascending Node	Longitude of Perihelion	Mean Longitude at Epoch
47 83	0 20562	7° 00′ 12″	47° 22′ 59″	76° 12′ 39″	165° 14′ 43″
34 99	0 00681	3 23 38	75 57 35	130 26 44	166 36 34
29 76	0 01674	0 00 00		101 33 53	99 51 02
24 11	0 09333	1 51 01	48 56 25	334 35 12	162 05 15
17 89	0 07653	10 36 56	80 45 39	149 26 12	223 19 21
24 64	0 22297	10 49 40	303 35 09	121 25 32	28 36 33
13 05	0 04837	1 18 28	99 38 24	13 02 01	125 18 37
9 64	0 05582	2 29 29	112 57 29	91 28 50	151 16 01
6 80	0 04710	0 46 22	73 35 27	169 22 07	329 20 35
5 43	0 00855	1 46 38	130 53 56	43 55 50	128 59 54

Mean Density		Axial Rotation	Inclination of Equator to Orbit	Oblateness	Mean Surface Gravity ⊕=1	Albedo	Stellar Magnitude at Mean Opposition	Velocity of Escape Km /Sec.
⊕=1	Water=1							
0 256	1 41	24^d 65 (equatorial)	7°10′.5	0 0000	27 89		−26 72	617 0
0 604	3 33	$27^d\ 7^h\ 43^m\ 11^s$ 5	6°40′ 7	0 0006	0 165	0 07	−12.55	2 4
0 70	3 8	$88^d.0$		0 00	0 27	0 07	0.16 *	3 6
0 88	4 86			0 00	0 85	0.59	−4.07 *	10.2
1 000	5.52	$23^h\ 56^m\ 4^s$ 09	23°26′ 59″	$\frac{1}{296}$	1.00	0 45?	−3 5 †	11.2
0 72	3.96	$24^h\ 37^m\ 22^s.58$	25° 10′	$\frac{1}{192}$	0 38	0 15	−1 85	5 0
0 6?	3 3?				0 037 ?	0.06	7.15	0.5?
0 6?	3.3?	$5^h\ 16^m$			0 001 ?		9.7	0.02?
0 242	1 34	$9^h 50^m$ to $9^h 55^m$	3° 6′.9	$\frac{1}{15\ 4}$	2 64	0.44	−2 23	60
0.13	0 71	$10^h 14^m$ to $10^h 38^m.5$	26° 44′.7	$\frac{1}{9.5}$	1 17	0 42	+0 89 to −0 18	36.
0 23	1.27	10^h 7	98°.0	$\frac{1}{14}$	0.92	0 45?	5.74	21.
0 29	1 58	15^h?	151°	$\frac{1}{40}$?	1.12	0.52?	7 65	23.

* At elongation. † As seen from sun.

THE SOLAR SYSTEM

Inc of Orbit to Planet's Equator or to "Proper Plane"*	Mean Inc of Orbit or Proper Plane to Planet's Orbit	Eccentricity	Stellar Mag at Mean Opposition	Diameter Km	Mass	
					Primary = 1	Moon = 1
28° 35′ to 18° 19′ †	5° 8′ 33″	0 05490	−12 3	3476	$\frac{1}{81\ 56}$	1 00
MARS						
0° 57′ 5	25° 19′ 6	0 017	11 5	15?		
1 44 0	24 14 7	0.003	13 0	8?		
JUPITER						
0° 27′ 3	3° 6′ 9	0 0028	13 0	160?		
0 1 .6	3 6 7	0 0000	5 5	3730	0 000042	1 09
0 28 1	3 5 8	0 0003	5 7	3150	0 000025	0 65
0 11 0	3 2 3	0 0015	5 1	5150	0 000081	2 10
0 15 2	2 42 .7	0.0075	6 3	5180	0 000022	0 58
180 53 ‡	28 45 (1900)	0 155	13 7	130?	Perturbations large	
243 0 ‡	27 58 (1900)	0 207	16	40?		
208 ‡	148 4 (1910)	0 378	16	25?	(1910–1916)	Perturbations enormous
31 ‡	156 (1914)	0 25	18	25?	(1914–1916)	
SATURN						
1° 36′ 5	26° 44′ 7	0 0190	12 1	650?	$\frac{1}{16\ 840\ 000}$	$\frac{1}{2120}$
0 1 .4	26 44 7	0 0001	11.6	800?	$\frac{1}{4\ 000\ 000}$?	$\frac{1}{520}$?
1 4 4	26 44 7	0 0000	10 5	1300?	$\frac{1}{921\ 500}$	$\frac{1}{110}$
0 4 0	26 44 7	0 0020	10.7	1200?	$\frac{1}{536\ 000}$	$\frac{1}{69}$
0 19 8	26 41 9	0 0009	10 0	1750?	$\frac{1}{250\ 000}$?	$\frac{1}{80}$?
0 16 9	26 7 1	0.0289	8.3	4200	$\frac{1}{4150}$	1.86
17′ to 56′	26 0 .0	0 1043	13.0	500?	$< \frac{1}{4\ 500\ 000}$	$< \frac{1}{600}$
13° 51′ 3	16 18 1	0 0284	10.1 to 11 9	1800?	$< \frac{1}{100\ 000}$	$< \frac{1}{18}$
148°.9 †	174 .7	0.1659	14.5	250?		
URANUS						
0°	97° 59′	0.007	15.2?	900?		
0	97 59	0.008	15 8?	700?		
0 0′	97 59	0.0023	14 0	1700?		
0 0	97 59	0 0010	14.2	1500?		
NEPTUNE						
20° †	139 49′ (1900)	0.000	13.6	5000?		

* See note at top of page vi. † To planet's equator ‡ Longitude of ascending node.

NAME INDEX